Fisher Info and Quantum Mechanics

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Wheeler's paradigm

- John Archibald Wheeler wrote the following: It is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer. It from bit. Otherwise put, every IT, every particle, every field of force, even the space time continuum itself, derives its function, its meaning, its very existence entirely, even if in some contexts indirectly, from the apparatus elicited answers to yes or no questions, binary choices, bits.
- It from bit symbolizes the idea that every item of the physical world has at bottom, a very deep bottom, in most instances, an immaterial source and explanation,
- that which we call reality arises in the last analysis from the posing of yesno questions and the registering of equipment-evoked responses; in short, that all things physical are information theoretic in origin and that this is a participatory universe.

Fisher information measure

- Last 15 years: physics' applications of Fisher's information measure (FIM), created in 1926. If associated with translations of a one dimensional observable x (probability density $\rho(x)$) is
- $$I_x = \int \mathrm{d}x\, \rho(x) \left(\frac{\partial \ln \rho(x)}{\partial x}\right)^2, \tag{1}$$
- $$I_x = \int \mathrm{d}x\, \rho(x)\, (\nabla \ln \rho(x))^2 \;\; or, \;\; setting \;\; \rho(x) = \psi(x)^2, \tag{2}$$
- $$I_x = 4 \int \mathrm{d}x\, [\nabla \psi(x)]^2. \tag{3}$$
- I_x obeys the so-called Cramer-Rao inequality

$$(\Delta x)^2 \geq I_x^{-1}, \text{involving variance :} \tag{4}$$

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int \mathrm{d}x\, \rho(x)\, x^2 - \left(\int \mathrm{d}x\, \rho(x)\, x\right)^2. \tag{5}$$

$S = -\int dx\, p(x) \ln p(x)$: MaxEnt's ingredients

- Lagrange multipliers λ_j
- the information quantifier, and
- "prior"expectation values $< A_j >$ $(j = 1, \ldots, M)$ (input information).
- For each j we have a pair $(\lambda_j\, ,\, A_j)$.
- Always a normalization Lagrange multiplier α that is Legendre conjugate to S.
- Reciprocity conjugate pairs $([S] - < A_j >)$ and $(\alpha\ -\ \lambda_j)$.
- $[S] = S(< A_1 >, \ldots, < A_M >)$ BUT $\alpha = \alpha(\lambda_1, \ldots, \lambda_M)$.
- Shannon's variation leads to an exponential function.
- $f_{ME}(x) = \exp\{-[\alpha + \sum_{i=1}^{M} \lambda_i A_i(x)]\}$.

More Fisher

- A MaxEnt-like program was successfully developed by us that replaces Shannon's information measure S by Fisher's one I. (local measure) in three relevant papers:
- 1) B. Frieden, A. Plastino, A. R. Plastino, B. Soffer, PRE **60** (1999) 48;
- 2) FPPS, PRE **66** (2002) 046128
- 3) S. P. Flego + FPPS, PRE **68** (2003) 016105.
- Instead of a FIXED-form PDF, a differential equation for the PDF!!
- Such diff. eq. is Schroedinger's!!

FPPS Fisher treatment: Part I

- Fisher Measure I reads, for PDF $f = \psi^2$:
  $I(\langle A_1 \rangle, \ldots, \langle A_M \rangle) = 4 \int dv \, [\nabla \psi(v)]^2$
- System specified by set of M physical quantities,
- $\mu_k = \langle A_k \rangle$ with $A_k = A_k(v)$.
- μ_k are prior knowledge, empirical information measured.
- $\langle A_k \rangle = \int dv \, A_k(v) \, f(v), \; k = 1, \ldots, M$.
- These mean values play here the role of extensive thermodynamical variables.

FPPS Fisher treatment: Part II

- $-\frac{1}{2}\nabla^2\psi - [\sum_{k=1}^{M}\frac{\lambda_k}{8}A_k]\psi = \frac{\alpha}{8}\psi$
- Schrödinger equation for a particle of unit mass moving in the potential
- $U = U(v) = -\frac{1}{8}\sum_{k=1}^{M}\lambda_k A_k(v)$
- Lagrange multiplier $(\alpha/8)$ plays the role of an energy eigenvalue $E = \alpha/8$.
- The λ_k are fixed by recourse to the available prior information.
- $\psi(v)$ is always real in the case in one dimensional scenarios, or for the ground state of a real potential in N dimensions.

Legendre Transform

- **Adrien-Marie Legendre**
  (September 1752 , January 1833).
- The Legendre transform is an operation that transforms one real-valued function of a real variable into another, maintaining constant its information content.
- The derivative of the function f becomes the argument to the function f_T.
- $\mathrm{f}_T(y) = xy - f(x);\ y = f'(x) \Rightarrow$ reciprocity. Own inverse.
- **Legendre ransformation is used to get from Lagrangians the Hamiltonian formulation of classical mechanics.**
- $E(S, V, N) - F(T, V, N) \Rightarrow\ -F = \frac{\partial E}{\partial S} S - E = TS - E$
  $-(\partial F/\partial T) = S$, $(\partial E/\partial S) = T$ Reciprocity Relations (RR).

$S-$MaxEnt reciprocity

- $f_{ME}(x) = \exp\{-[\alpha + \sum_{i=1}^{M} \lambda_i A_i(x)]\}$.
- $\alpha(\vec{\lambda}) = \ln\{\int dx\, [\exp(-\sum_{i=1}^{M} \lambda_i A_i(x))]\}$,
- $\boxed{[\partial\alpha(\lambda_1, \ldots, \lambda_M)/\partial\lambda_j] = -\langle A_j\rangle,}$
- $S = \alpha + \sum_{i=1}^{M} \lambda_i \langle A_i\rangle$,
- $dS = \sum_{i=1}^{M} \lambda_i\, d\langle A_i\rangle$
- $\boxed{\Rightarrow \quad (\partial S(\langle A_1\rangle, \ldots, \langle A_M\rangle)/\partial\langle A_i\rangle) = \lambda_i}$
- The reciprocity relations (in blue) are a manifestation of the Legendre invariant structure of thermodynamics and its most salient structural mathematical feature.

Fisher reciprocity relations

- $$I(\langle \vec{A_k} \rangle) = \alpha + \sum_{k=1}^{M} \lambda_k \langle A_k \rangle .$$
- $$\alpha(\vec{\lambda_k}) = I(\langle \vec{A_k} \rangle) - \sum_{k=1}^{M} \lambda_k \langle A_k \rangle .$$
- $$\frac{\partial \alpha}{\partial \lambda_i} = -\langle A_i \rangle, \quad \lambda_k = \frac{\partial I}{\partial \langle A_k \rangle}.$$
- Recip. Relat. a Fisher-feature $\Rightarrow$ Fisher-thermodynamics. F. Pennini, A. Plastino, *Reciprocity relations between ordinary temperature and Fisher temperature*, PRE **71** (2005) 047102.

Hellmann-Feynman theorem and reciprocity

- Fisher Information, Hellmann-Feynman Theorem, and Jaynes' Reciprocity Relations, Annals of Physics **326** (2011) 2533.
- **Feynamn-Hellmann theorem (HFT)**: how perturbations in an operator affect the operator's eigenvalues.
- HFT theorem assures that an eigenvalue $E_i(b)$ of $H(b)$ of eigenvector $\psi_i(b)$ varies with respect to b according to:
- $$\frac{\partial E_i}{\partial b} = \langle \psi_i | \frac{\partial H}{\partial b} | \psi_i \rangle$$
- *We showed that the above relation is just a reciprocity relation* if we express the potential in a series-expansion
- $U(x) = \sum_n c_n x^n$, and regard c_n as parameters.

Virial-HF theorems are traditional properties of SE

- These properties lead to the SE's Legendre structure by themselves!
- QUANTUM CONNECTION
  Legendre-transform structure derived from quantum theorems
  S.P. Flego, A. Plastino, A.R. Plastino, Physica A **390** (2011) 2276.
- VIRIAL THEOREM: For any stationary state of $H = T + U$
  $\langle -(\hbar^2/m)\,\vec{\nabla}\,\rangle_\psi = \langle \vec{x}.\vec{\nabla}U(\vec{x})\rangle_\psi$.
- Helmann-Feynman Theorem: $(\partial E_i/\partial b) = \langle \psi_i|(\partial H/\partial b)|\psi_i\rangle$.
- $-\frac{m}{2}\nabla^2\psi + U(x)\psi = E\psi$. Ordinary Schr. Eq. with assoc. I_ψ

- Assume that our prior information is that of the mean value of a potential function $V(x)$ in phase-space. Fisher variational problem becomes
- $$\delta \left\{ I + \lambda_0 \int \frac{dxdp}{h}\, f(x,p) + \lambda \int \frac{dxdp}{h}\, V(x) f(x,p) \right\} = 0. \quad (6)$$
- Schroedinger Eq. particle moves in potential well $V(x)$.
- *Trajectories in the coordinate-subspace of classical phase space that minimize Fisher's measure are identical to their quantum mechanical counterparts.*
- Out of the infinitude of phase-space orbits, minimizing FIM selects out just the quantum orbit subset. F. Olivares, A. Plastino, B. H. Soffer, *Quantum trajectories emerging from classical phase space*, Physica A **390** (2011) 1926-1930.

- Non-interacting classical particles of mass m with coordinates $\mathbf{q} = (\mathbf{r}, \mathbf{p})$, where $m d\mathbf{r}/dt = \mathbf{p}$. In extremizing FIM we constrain the normalization of $\rho(\mathbf{r})$ and $\eta(\mathbf{p})$ to the total number of particles N and to 1, respectively.
- Translational invariance is described by family of distributions $F(\mathbf{r}', \mathbf{p}')$ whose form does not change under the transformations $\mathbf{r}' = \mathbf{r} - \theta_r$ and $\mathbf{p}' = \mathbf{p} - \theta_p$. The density can obviously be factorized in the fashion $F(\mathbf{r}, \mathbf{p}) = \rho(\mathbf{r})\eta(\mathbf{p}) \;\Rightarrow I = I_r + I_p$
- $$\begin{aligned} I_r &= c_r \int d^D\mathbf{r}\, \rho(\mathbf{r})\, |\nabla_r \ln \rho(\mathbf{r})|^2 \\ I_p &= c_p \int d^D\mathbf{p}\, \eta(\mathbf{p})\, |\nabla_p \ln \eta(\mathbf{p})|^2 . \end{aligned} \tag{7}$$

- Penalize infinite values particle momentum with variance constraint for $\eta(\mathbf{p})$ to an empirically value

$$\int d^D\mathbf{p}\; \eta(\mathbf{p})(\mathbf{p}-\overline{\mathbf{p}})^2 = D\sigma_p^2 = Dmk_BT, \tag{8}$$

where $\overline{\mathbf{p}}$ is the mean value of $\mathbf{p}$.

- Virial Theorem: variance related to temperature T as $\sigma_p^2 = mk_BT$, k_B Boltzmann const. Below μ, λ and ν are Lagrange multipliers.
- $$\delta\left\{c_r\int d^D\mathbf{r}\;\rho\,|\nabla_r \ln\rho|^2 + \mu\int d^D\mathbf{r}\;\rho\right\} = 0 \tag{9}$$

and

$$\delta\left\{c_p\int d^D\mathbf{p}\;\eta\,|\nabla_p \ln\eta|^2 + \lambda\int d^D\mathbf{p}\;\eta(\mathbf{p}-\overline{\mathbf{p}})^2 + \nu\int d^D\mathbf{p}\;\eta\right\} = 0. \tag{10}$$

- $$\rho(\mathbf{r})\eta(\mathbf{p}).\ \textit{Vary w.r.t. } \rho \textit{ gives } \left[-4\nabla_r^2 + \mu'\right]\Psi(\mathbf{r}) = 0, \quad (11)$$

  where $\mu' = \mu/c_r$. To fix the boundary conditions, we first assume that the N particles are confined in a box of volume V, and next we take the thermodynamic limit $N, V \to \infty$ with N/V finite. The equilibrium state compatible with this limit corresponds to the ground state solution ($\mu' = 0$), which is the uniform density $\rho(\mathbf{r}) = N/V$.
- Introducing $\eta(\mathbf{p}) = \Phi^2(\mathbf{p})$ and varying with respect to Φ leads to the quantum harmonic oscillator-like equation

  $$\left[-4\nabla_p^2 + \lambda'(\mathbf{p} - \overline{\mathbf{p}})^2 + \nu'\right]\Phi(\mathbf{p}) = 0, \qquad (12)$$

  where $\lambda' = \lambda/c_p$ and $\nu' = \nu/c_p$. The equilibrium configuration corresponds to the ground state solution, which is now a gaussian distribution.

Fisher treatment of the $D-$dimensional ideal gas

- Using (8) to identify $|\lambda'|^{-1/2} = \sigma_p^2$ we get the Maxwell-Boltzmann distribution, which leads to a density distribution in configuration space of the form

$$f(\mathbf{r}, \mathbf{p}) = \frac{N}{V} \frac{\exp\left[-(\mathbf{p} - \overline{\mathbf{p}})^2/2\sigma_p^2\right]}{(2\pi\sigma_p^2)^{D/2}}. \tag{13}$$

$\mathcal{V}$ is elementary phase-space Vol. Total microstate Nr. is $Z = N!\mathcal{V}^{DN} \prod_{i=1}^{N} F_1(\mathbf{r}_i, \mathbf{p}_i)$, where $F_1 = F/N$ is the single-particle distr. $N!$ counts all possible permutations for distinguishable particles. $S = -k_B \ln Z$ becomes, using Stirling, well-known:

$$S = Nk_B \left\{ \ln \frac{V}{N} \left(\frac{2\pi\sigma_p^2}{\mathcal{V}^2} \right)^{D/2} + \frac{2+D}{2} \right\}, \tag{14}$$

- Because of the nature of the systems to be addressed we consider now a one-dimensional system with a physical parameter θ and a discrete coordinate $k = k_1, k_2, \ldots, k_i, \ldots$ where $k_{i+1} - k_i = \Delta k$ for a certain value of the interval Δk. This scenario arises, for instance, in the case of nuclear multifragmentation, the abundances of genes, the frequency of words, scientific collaboration networks, the Internet traffic, Linux packages links, electoral results, urban agglomerations, firm sizes, etc.
- In the continuous limit ($\Delta k \to dk$), the Fisher information measure is cast as

$$I(F) = c_k \int_{k_1}^{\infty} dkF(k|\theta) \left| \frac{\partial}{\partial \theta} \ln F(k|\theta) \right|^2 . \qquad (15)$$

Scale invariance II

- Instead of using translation invariance, we appeal to scaling invariance
- F. Pennini, A. Plastino, B. H. Soffer, and C. Vignat, Phys. Let. A 373 (2009) 817.
- so that we can anticipate some new physics. All members of the family $F(k/\theta)$ possess identical shape —there are no characteristic size, length or frequency for the observable k— namely $dkF(k/\theta) = dk'F(k')$ under the transformation $k' = k/\theta$. To deal with this new symmetry it is convenient to change to the new coordinate $u = \ln k$ and parameter $\Theta = \ln \theta$. Why?

- Because then the scale invariance becomes again translational invariance, and we are entitled to use essential result, namely, that MFI leads to a Schroedinger-like equation. Note that the new coordinate $u' = \ln k'$ transforms as $u' = u - \Theta$. Defining $f(u) = F(e^u)$ and taking into account the fact that the Jacobian of the transformation is $|dx/du| = e^u$ and $\partial/\partial\theta = e^{-\Theta}\partial/\partial\Theta$, the Fisher information measure acquires now the form

$$I(F) = c_k e^{-2\Theta} \int_{u_1}^{\infty} du\ e^u f(u) \left| \frac{\partial \ln f(u)}{\partial u} \right|^2 , \qquad (16)$$

where $u_1 = \ln k_1$, and the factor $e^{-2\Theta}$ guaranties the invariance of the associated Cramer-Rao inequality as shown in PLA Vignat.

More Scale invariance

- Scale invariance is a feature of objects or laws that do not change if scales of length, energy, or other variables, are multiplied by a common factor. The technical term for this transformation is a dilatation (also known as dilation),
- One-dimensional system with dynamical coordinates $\mathbf{q} = (k, v)$ where $dk/d\tau = v$, with τ the time variable. We define k as a **discrete** coordinate, i.e. $k = k_1, k_2, \ldots, k_M$, where $k_i = i\Delta k$ and $M \gg 1$, is the total number of bins of width Δk in our system.
- In order to address the scale-invariance behaviour of k we change variables passing to new coordinates $u = \ln k$ and $w = du/dt$.
- u and w are canonically conjugated and uncorrelated. This assumption immediately leads to proportional growth since

$$dk/dt = v = kw. \tag{17}$$

- For constant w Eq. $dk/dt = v = kw.$ yields exponential growth $k = k_0 e^{wt}$, which represents uniform linear motion in u, that is, $u = wt + u_0$, with $u_0 = \ln k_0$.
- Scale transformation $k' = k/\theta_k$ leaves invariant the coordinate w, whereas the coordinate u transforms translationally as $u' = u - \Theta_k$, where $\Theta_k = \ln \theta_k$. Physics scale independent! System Translat. invariant with respect to the coordinates u and w, so that distribution of physical elements described by monoparametric translation familiesas above.
- By analogy with the IG, we will call our system a "scale-free ideal gas" (SFIG), i.e., a system of N non-interacting elements.

- Taking into account that i) u and w are canonical and uncorrelated the density distribution can be factorized as $f(u,w) = g(u)h(w)$,and ii) that the Jacobian for our change of variables is $dkdv = e^{2u}dudw$, the information measure $I = I_u + I_w$ can be obtained in the continuous limit as

$$\begin{aligned} I_u = &\quad c_u \int_{\Omega} du\; e^{2u} g(u) \left| \frac{\partial \ln g(u)}{\partial u} \right|^2 \\ I_w = &\quad c_w \int_{-\infty}^{\infty} dw\; h(w) \left| \frac{\partial \ln h(w)}{\partial w} \right|^2 , \end{aligned} \qquad (18)$$

where $\Omega = \ln(k_M/k_1) = \ln M$ is the volume defined in u space.

Fisher-Scroedinger treatment of the the scale-free ideal gas

- $$\int_\Omega du\, e^{2u} g(u) = N, \qquad \int_{-\infty}^{\infty} dw\, h(w) = 1. \qquad (19)$$

  We penalize infinite values for w with a constraint on the variance of $h(w)$ to a given measured value

  $$\int_{-\infty}^{\infty} dw\, h(w)(w - \overline{w})^2 = \sigma_w^2, \qquad (20)$$

  where $\overline{w}$ is the average growth. The variation yields

  $$\delta \left\{ c_u \int_\Omega du\, e^{2u} g \left| \frac{\partial \ln g}{\partial u} \right|^2 + \mu \int_\Omega du\, e^{2u} g \right\} = 0 \qquad (21)$$

  and

  $$\delta \left\{ c_w \int_{-\infty}^{\infty} dw\, h \left| \frac{\partial \ln h}{\partial w} \right|^2 + \lambda \int_{-\infty}^{\infty} dw\, h (w - \overline{w})^2 + \nu \int_{-\infty}^{\infty} dw\, h \right\} \qquad (22)$$

  where μ, λ and ν are Lagrange multipliers.

Fisher-Scroedinger treatment of the the scale-free ideal gas II

- Introducing $g(u) = e^{-2u}\Psi^2(u)$, and varying with respect to Ψ leads, as is always the case with the MFI , to the Schroedinger-like equation

$$\left[-4\frac{\partial^2}{\partial u^2} + 4 + \mu'\right]\Psi(u) = 0, \qquad (23)$$

where $\mu' = \mu/c_u$. Analogously to the IG, we impose solutions compatible with a finite normalization of $g(u)$ in the thermodynamic limit $N, \Omega \to \infty$ with $N/\Omega = \rho_0$ finite, where ρ_0 is defined as the *bulk density*. Solutions compatible with the normalization of (19) are given by $\Psi(u) = A_\alpha e^{-\alpha u/2}$, where A_α is the normalization constant and $\alpha = \sqrt{4+\mu'}$. In this general case, the density distribution as a function of k takes the form of a power law: $g(u)_\alpha(\ln k) = A^2/k^{2+\alpha}$. The equilibrium is always defined for the MFI as the ground state solution, which corresponds to the lowest allowed value $\alpha = 0$.

Fisher-Scroedinger treatment of the the scale-free ideal gas III

- Introducing now $h(w) = \Phi^2(w)$ and varying with respect to Φ leads to the quantum harmonic oscillator-like equation

$$\left[-4\frac{\partial^2}{\partial w^2} + \lambda'(w - \overline{w})^2 + \nu'\right]\Phi(w) = 0, \qquad (24)$$

where $\lambda' = \lambda/c_w$ and $\nu' = \nu/c_w$. The equilibrium configuration corresponds to the ground state solution, which is now a Gaussian distribution. Using (20) to identify $|\lambda'|^{-1/2} = \sigma_w^2$ we get the Maxwell-Boltzmann distribution

$$h(w) = \frac{\exp\left[-(w - \overline{w})^2/2\sigma_w^2\right]}{\sqrt{2\pi}\sigma_w}. \qquad (25)$$

The density distribution in configuration space $F(k, v)dkdv = f(u, w)e^{2u}dudw$ is then

$$F(k, v) = \frac{N}{\Omega k^2}\frac{\exp\left[-(v/k - \overline{w})^2/2\sigma_w^2\right]}{\sqrt{2\pi}\sigma_w}. \qquad (26)$$

Fisher-Scroedinger treatment of the the scale-free ideal gas IV

- If we define $\hbar = \Delta k^2/\Delta\tau$ as the elementary volume in phase space, where $\Delta\tau$ is the time element, the total number of microstates is $Z = N!\hbar^N \prod_{i=1}^N F_1(k_i, v_i)$, where $F_1 = F/N$ is the monoparticular distribution function and $N!$ counts all possible permutations for distinguishable elements. The entropy equation of state $S = -\kappa \ln Z$ reads

$$S = N\kappa \left\{ \ln \frac{\Omega}{N} \frac{\sqrt{2\pi}\sigma_w}{H'} + \frac{3}{2} \right\}, \tag{27}$$

  where κ is a constant that accounts for dimensionality and $H' = \hbar/(k_M k_1) = \hbar/(M\Delta k^2) = 1/(M\Delta\tau)$. Remarkably, this expression has the same form as the one-dimensional IG ($D = 1$ in (14)); instead of the thermodynamical variables (N, V, T), here we deal with the variables (N, Ω, σ_w), which make the entropy scale-invariant as well.

- The total density distribution for k is obtained integrating for all v the density distribution in configuration space. Accordingly, from (26) we get

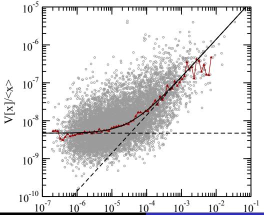
$$F(k) = \int dv F(k,v) = \frac{N}{\Omega}\frac{1}{k} = \frac{\rho_0}{k}. \tag{28}$$

  It can be show that it is just a uniform density distribution in u space of the bulk density: $F(k)dk = f(u)e^u du = N/\Omega du = \rho_0 du$.
- Represent empirical data with **rank-plot**. The jth system's element represented by its size, length or frequency k_j vs. its rank, sorted from the largest to the smallest one. This process just renders the inverse function of the ensuing cumulative distribution, normalized to the number of elements. We call r the rank that ranges from 1 to N. For large N, the density distribution (28) correspond to an exponential rank-size distribution

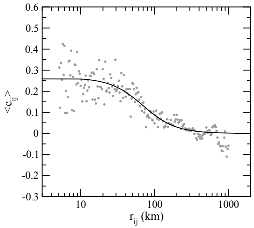
$$k(r) = k_M \exp\left[-\frac{r-1}{\rho_0}\right]. \tag{29}$$

Social examples of scale-free ideal gases

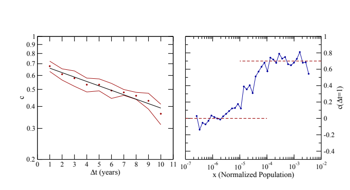
- Rank-size distribution of the cities of the province of Huelva, Spain (2008), sorted from largest to smallest, compared with the result of a simulation with Brownian walkers (green squares).



- Rank-plot of the 2008 General Elections results in Spain.



- Rank-plot of the 2005 General Elections results in the United Kingdom. (Red dots and fit to (29)).



Digital

- Digital physics is a collection of theoretical perspectives based on the premise that the universe is, at heart, describable by information, and is therefore computable.
- Thus, the universe can be conceived of as the output of a program in a vast, digital computation device.
- Digital physics is grounded in one or more of the following hypotheses; listed in order of decreasing strength. The universe, or reality:

Digital II

- is essentially informational (although not every informational ontology needs to be digital)
- is essentially computable
- can be described digitally
- is in essence digital
- is itself a computer
- is the output of a simulated reality exercise

Digital IV

- Every computer must be compatible with the principles of information theory, statistical thermodynamics, and quantum mechanics. A fundamental link among these fields was proposed by Edwin Jaynes in two seminal 1957 papers.
- The hypothesis that the universe is a digital computer was pioneered by Konrad Zuse in his book Calculating Space. The term digital physics was first employed by Edward Fredkin, who later came to prefer the term digital philosophy. Others who have modeled the universe as a giant computer include Stephen Wolfram, and Nobel laureate Gerard ’t Hooft.
- These authors hold that the apparently probabilistic nature of quantum physics is not necessarily incompatible with the notion of computability. Support from Seth Lloyd and David Deutsch.