A Fuzzy Set Approach to Poverty Measurement*

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This version: June 23, 2005

Abstract

This paper postulates two poverty indices based on a fuzzyfication of the poverty line approach and shows that they satisfy some of the usual axioms in the poverty line literature. It also shows that the *headcount ratio* is a particular case of a poverty measure based on fuzzy sets.
Finally this paper postulates that fuzzy version of poverty measures will not satisfy the transfer axiom.

JEL Classification: I32
Keywords: Poverty Measurement

1 Introduction

The measurement of poverty is an issue prone to debate: about the definition of poverty, or about the choice between welfare measurement based on consumption or on income, or between a unidimentional (like poverty lines) or a multidimentional approach, etc. Nonetheless, what is generally agreed is that the process of transition from poverty to a well-being state is gradual.

Fuzzy set theory gives us convenient tools to model this process as well as to represent ambiguous linguistic concepts. In this paper, we propose two poverty measures based on the poverty line approach and apply fuzzy sets to satisfy the conditions requested for such a measure.

In section 2, we make a revision of the traditional poverty indexes. After an exposition of the axiomatic framework that characterizes such indexes, we detail the most common poverty measures and show which axioms they satisfy. In addition, we make a brief review of fuzzy notions and the basic operations in this theory. Finally, we survey other papers that treat poverty measures by means of fuzzy sets.

In section 3, we present the *membership function* and we emphasize on its principal characteristics. Once specified this function, we propose two indexes of poverty measurement: on one hand an index that considers the α-cuts and on the other hand an index based on the *membership function.* As an illustrative journey we estimate the *membership function* and our indexes comparing them with the traditional ones using a household survey for Argentina. Later, we evaluate the satisfaction of the axioms listed in section 2 by each of both indexes. As a result we found that the *headcount ratio* is a particular case of a fuzzy index.

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* For helpful comments we are very grateful to Fernando Tohmé. The usual disclaimer applies.
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2 Poverty Measures and Fuzzy Set Theory

2.1 Poverty and Axioms

Three issues are important referred to the poverty phenomena: its concept, the identification of poor people and the way to quantify poor individuals and then obtain an aggregated measurement for the society.

We will consider poverty as the state of deprivation in which an individual can not enjoy a "normalized standard of living". Such standard of living is defined by a minimal level of income that allows an individual to buy an appropriate combination of goods in order to satisfy the basic needs of a human being.

Closely related with the concept of poverty is the issue of the method of identification. There are two ways to settle down who must be considered poor:

- Direct methods: in this kind of analysis the living conditions of the individuals are directly evaluated. The most popular version is the Unsatisfied Basic Needs method (UBN).

- Indirect Methods: these poverty analyses evaluate the consumption power of the individuals. This method is popular for using poverty lines since they establish the minimal income required to sustain an adequate standard of living. Then, all the households whose incomes are below the poverty line are labeled as poor.

The UBN method works counting the households that have not satisfied a series of previously established necessities which will be considered poor. The UBN is very strict, so a family is deemed poor even if just only one necessity is lacking. This measure presents many restrictions: it can not be applied to do intertemporal or geographical comparisons and it is sensitive to the number of unsatisfied needs required to be considered poor. Nevertheless, this approach is useful when the objective is the construction of geographic poverty maps with on extensive characterization.

In this paper, instead we will focus on the poverty line method of poverty measurement, which has well known capabilities, like the possibility of comparing between different temporal surveys, or even, between different societies.

The benchmark axioms that a poverty index must satisfy appear in Sen (1976) and were later extended by other authors.

Monotonicity Axiom: Given other things, a reduction in the income of a poor household must increase the poverty measure (Sen, 1976).

Transfer Axiom: Given other things, a pure transfer of income from a poor household to any other household that is richer must increase the poverty measure (Sen, 1984).

Subgroup Monotonicity Axiom: Suppose that the population is divided into m collections of households j=1,…,m with ordered income vectors $y^{(j)}$ and population sizes n_j. Let $\hat{y}$ be a vector of incomes obtained from y by changing the incomes in subgroup j from $y^{(j)}$ to $\hat{y}^{(j)}$, where n_j is unchanged. If $\hat{y}^{(j)}$ has more poverty than $y^{(j)}$, then $\hat{y}$ must also have a higher level of poverty than y (Foster et. al., 1984).

In addition, there are two properties which should be considered about a poverty measure.

Invariance to Scale Axiom: The poverty measure must be homogeneous of degree zero in incomes.

Poverty Line Level Effect Axiom (z-level effect): If the poverty line is set in z_0 below z_1 an alternative poverty line, the weight assigned in the poverty measure to each monetary unit of the household incomes has to be higher in z_0 than in the poverty line z_1.

2.2 Poverty Measures based on Poverty Line

Now we consider some of the different indexes created to measure poverty. Then we will evaluate which axioms they satisfy and which not.

The poverty line approach usually considers that an individual $i \in [0, N]$, where *N* is the total number of individuals in the population, is poor if he has an income that falls below a certain amount of money *z*. In other words, consider a set X that is our population of individual incomes and $x_i \in X$ the income associated with agent *i*, therefore the set $P = \{i | x_i < z\}$ is the collection of agents considered poor. We also define q where $q = |P|$.

2.2.1 Headcount Ratio

This index directly gives us the proportion of individuals with income below the poverty line.

$$H = \frac{q}{N} \tag{1}$$

We can show that this measure does not satisfy the monotonicity axiom. Therefore, the headcount ratio does not provide information about the depth of poverty. Nevertheless, this index satisfies Foster's et. al. (1984) axiom and thus is decomposable. In addition, this measure is homogeneous of degree zero in incomes and it fails to comply with the z-level effect axiom.

2.2.2 Income Gap ratio:

$$I = \sum_{i \in P} \frac{g_i}{qz} \tag{2}$$

where $g_i = z - x_i$ is the income shortfall of the ith household.

The income gap ratio measures the mean shortfall of the poor income households. As we can check it does not give us information about how many people are poor, but it is useful to know the depth of poor incomes.

As x_i falls, g_i grows up as well as the index. It means that this measure satisfies the monotonicity axiom. We can show that this index also satisfies subgroup monotonicity but does not satisfy the transfer axiom. Additionally, since incomes are normalized then the index is invariant to scale. Finally, this measure assigns equal weights to all individuals then it does not satisfy the z-level effect axiom.

2.2.3 Foster – Greer – Thorbecke Index (FGT)

The FGT index transforms the income gap ratio in order to allow an unequal ponderation of the poorest families in the aggregation.

$$FGT_\alpha = \frac{1}{N}\sum_{i=1}^{q}\left(\frac{g_i}{z}\right)^\alpha \tag{3}$$

The α parameter is a measure of poverty aversion: a larger α gives greater emphasis to the poorest individuals. The FGT measure is more general that the previous ones. We can see that when α =0 the FGT is the headcount ratio

$$FGT_0 = \frac{1}{N}\sum_{i=1}^{q}1 = \frac{q}{N} \tag{4}$$

and when α =1 the FGT is the income gap ratio

$$FGT_1 = \frac{1}{N}\sum_{i=1}^{q}\left(\frac{g_i}{z}\right) \tag{5}$$

Finally, when α = 2 we get an alternative to the Sen poverty index (1976) in which the poor are weighted up by their g_i.

$$FGT_2 = \sum_{i=1}^{q}\left(\frac{g_i}{Nz}\right)\left(\frac{g_i}{z}\right) = \frac{1}{Nz^2}\sum_{i=1}^{q}g_i^2 \tag{6}$$

Foster et. al. (1984) show that this index satisfies the monotonicity axiom when α >0, the transfer axiom when α >1 and subgroup monotonicity axiom for any value of α. Finally, we can show that the FGT satisfies the invariance to scale axiom and z-level effect axiom.

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2.3 Fuzzy Set Theory

"A **classical** (crisp) **set** is normally defined as a collection of elements or objects $x \in X$ which can be finite, countable, or overcountable. Each single element can either belong to or not belong to a set *A*, $A \subseteq X$. In the former case the statement "*x* belongs to *A*" is true, whereas in the latter case this statement is false" (Zimmermann, 1991).

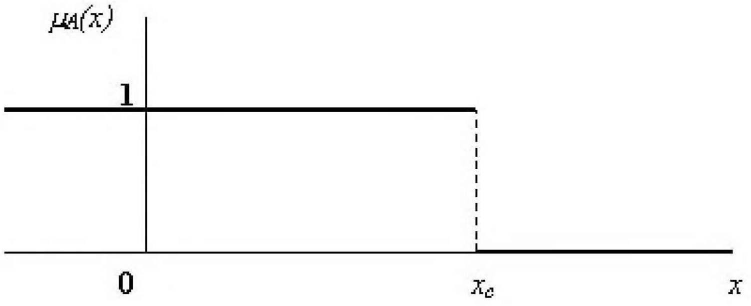
We can classify an element as a member or nonmember of a given set *B* by a *membership or characteristic function* such that

$$\forall x \in X : \mu_B(x) = \begin{cases} 1 & if \quad x \in B \\ 0 & if \quad x \notin B \end{cases} \tag{7}$$

"Thus, the $\mu_B(x)$ function maps elements of the universal set to the set containing 0 and 1. This can be indicated by $\mu_B : X \to \{0,1\}$" (Klir and Folger, 1988).

This function can also be defined for a continuous variable, for example the set $A = \{x \in \Re : x \le x_c\}$ where x_c is a critical value previously fixed. This is shown in figure 1.

FIGURE 1



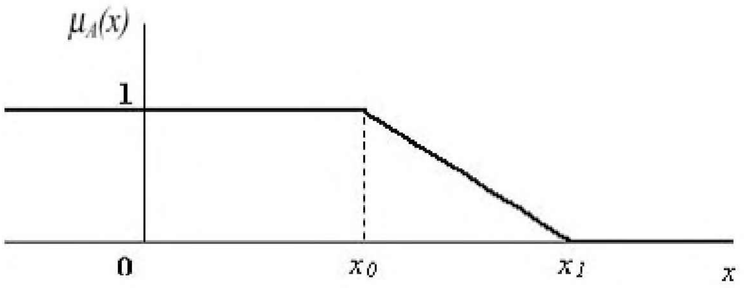
In reality "…one does not directly meet sets with a crisp "borderline", but quite often it seems that there exists something like a gradual transition between membership and non-membership" (Bandemer and Gottwald, 1999). In some discussions, the elements do not belong completely to a set. In such cases the collection of elements with a membership value between [0,1] is called a fuzzy set. The gradual transition mentioned above is represented by a continuum *membership function* μ_A which has the form: $\mu_A : X \to [0,1]$.

The *membership function* can be represented by many types of functions. The figure 2 shows the more typical case where μ_A is assumed linear.

The corresponding expression for this *membership function* is

$$\forall x \in X : \mu_A(x) = \begin{cases} 1 & \text{if} \quad x \leq x_0 \\ \dfrac{x_1 - x}{x_1 - x_0} & \text{if} \quad x_0 \leq x \leq x_1 \\ 0 & \text{if} \quad x > x_1 \end{cases} \qquad (8)$$

FIGURE 2



A fuzzy set is described by its elements and their membership value: considering the general case $A = \{(x, \mu_A(x)), x \in X\}$.

We can also define basic operations which are taken from classical set theory and are applied to fuzzy sets theory:

<u>*Complement*</u>: the complement $\overline{A}$ of *A* is defined by the *membership function* $\forall x \in X, \mu_{\overline{A}}(x) = 1 - \mu_A(x)$ (Dubois and Prade, 1980).

The classical union ($\cup$) or intersection ($\cap$) of regular subsets of *X* can be extended formulating the *membership function* for each one:

<u>Union</u>: $A \bigcup B = \{(x, \mu_{A \bigcup B}(x)), x \in X \wedge \mu_{A \bigcup B}(x) = \max\ (\mu_A(x), \mu_B(x))\}$ (Dubois and Prade, 1980).

<u>Intersection</u>: $A \bigcap B = \{(x, \mu_{A \bigcap B}(x)), x \in X \wedge \mu_{A \bigcap B}(x) = \min\ (\mu_A(x), \mu_B(x))\}$ (Dubois and Prade, 1980).

In addition to these operations there are also useful tools to model other cases with this mathematical approach. The most popular are:

<u>*α-cuts*</u>: The crisp set of elements that belong to a fuzzy set *A* at least to a membership value α is called the α-cut set, i.e. $A_\alpha = \{x \in X, \mu_A(x) \geq \alpha\}$.

Given the α-cuts we can define again the fuzzy set *A* by the decomposition theorem:

$$A = \bigcup_{\alpha \in [0,1]} A_\alpha \qquad (9)$$

<u>Scalar Cardinality</u>: The scalar cardinality of a fuzzy set A defined on a finite universal set *X* is $|A| = \sum_{x \in X} \mu_A(x)$.

As we will see later, some of the operations mentioned here will be applied later in order to derive a poverty measure based on fuzzy sets. But first, let us review some earlier works applying fuzzy sets to poverty measurement.

2.4 Other Applications of Fuzzy Set Theory to the Study of Poverty

In this section we summarize three works which are representative of the great variety of papers in which poverty is measured using fuzzy set theory. The most remarkable difference with our approach is that all of them elaborate a multidimensional index where the final measurement consist in a weighted aggregation of all the proposed dimensions.

The paper of Barán et. al. (1999) tries to represent the duality of poverty when analyzes the poverty phenomena in the Paraguayan economy. They say that the other face of poverty is welfare, and since both concepts are diffuse then nothing better than a fuzzy sets approach to work with them. With the goal of having a measure expressing up the depth of poverty, they include as fuzzy variables some economic, cultural, social and environmental factors which represent a parametric style of living.

Each fuzzy variable is associated to a linear *membership function* like the one showed in figure 2. Then they aggregate by the weighted sum of the whole number of variables. The weights were fixed exogenously by external specialists.

We consider that the chosen *membership function* is inappropriate because working with linear expressions implies to exogenously specify the critical value x_0 and x_1 in (8).

Second, since the weights are defined by the opinion of experts, it becomes hard to make comparisons between societies because the studies could be based on different weight criteria.

Finally, the paper defines the welfare index as the result of the complement operation applied to the poverty set. This measure is not the better option to measure welfare. A valid welfare index must be based on the population preferences or directly by the individual consumptions. (Lambert, 1993; Deaton, 1997).

Garcia et. al. (1998) define poverty based on the concept of minimal capacities of Sen. Nevertheless, they do not define a specific *membership function* in the article. As in Barán et. al. (1999) they suppose that the weights in the weighted sum of all the variables have to be specified by experts. Again, is seems arbitrary to include the weights in such a way.

The issue of the weights is crucial for Miceli (1998). In his work on Swiss poverty, he calculated the weights taking a logarithmic transformation of the average fuzzy proportion of deprived individuals. In contrast with this improved design, a linear *membership function* was again chosen. Since, the x_0 and x_1 in (8) were selected arbitrarily, the author indicates that a sensitive analysis must be run to check out that results do not depend critically on them.

3 Poverty Measures Based on Fuzzy Set Theory

3.1 Poverty as a Fuzzy Set

The traditional view supposes that the set P is a crisp subset of the population. Since the concept of poverty has an important degree of ambiguity, it seems reasonable to consider P as a fuzzy set and to propose two types of indices based on a fuzzy notion of the poverty line approach.

In this view an agent *i* will have a certain grade of association to P based on x_i and z. Relaxing, in this manner, the fact a person who earns one cent above z is not considered poor while another who earns one cent less than z is classified as poor. Also this analysis is in line with the consensus on that the transition from poverty to a non-poverty state happens rather gradually. In the next section, the paper shows that this grade of association to P is summarized in the *membership function*, which has parameters the grade of allowed fuzziness θ and the poverty line z. The two types of indices that we present in subsequent sections are based on this *membership function*.

3.2 Membership Function

A fuzzy set, in this case the poverty set P, is characterized by a *membership function* $\mu(\cdot)$ mapping the elements of a domain space, individual incomes, to the unit interval [0,1]. That is $\mu : X \to [0,1]$.

The *membership function* we propose for poverty measurement is:

$$\mu(x) = \frac{1}{1 + (10^{-\theta})^{\frac{-x}{z}+1}} \tag{10}$$

where: θ is a fuzziness parameter $\theta \in [0, \infty)$, *x* is the individual income and *z* is the poverty line. When $\theta \to 0$ the degree of fuzziness reaches a higher level for the *membership function*. Therefore, the extreme case is when $\theta = 0$ for which the *membership function* is going to assign the same membership grade (0.5) to each agent in the population. On the other hand, when $\theta \to \infty$ the degree of fuzziness considered is cero and the *membership function* is going to assign 1 to agents that have an income below *z* and cero to agents that have an income above *z*.

To illustrate the *membership function* we have computed it using the *Encuesta Permanente de Hogares* (EPH) a household survey carried out in Gran Buenos Aires (Argentina) in May of 2003. In the process of computing this function we use the adjustments for *adult equivalents.* Figure 3 shows the *membership function* specifying $\theta = 10$ and *z*=232.28, and figure 4 shows the *membership function* specifying $\theta = 2$ and *z*=232.28.

FIGURE 3

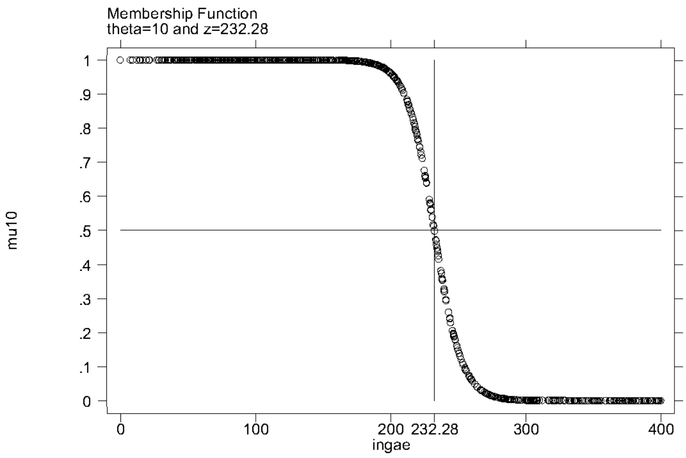
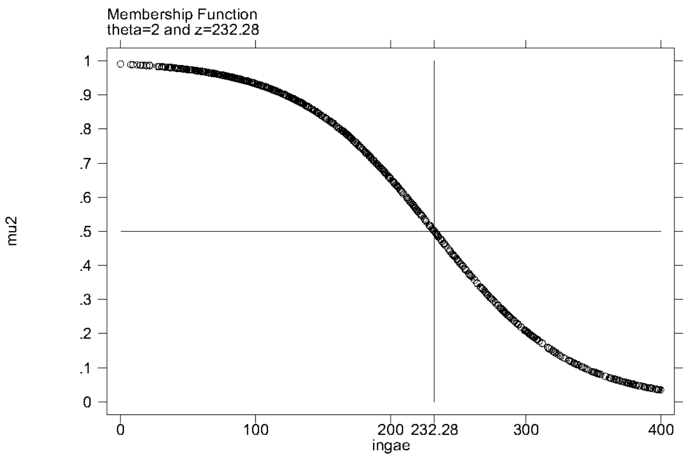


FIGURE 4



This function has some properties that are important to mention because they translate into the two types of poverty measures proposed below.

Proposition 1. $\mu(\cdot)$ is invariant to scale.

Proof: $$\frac{1}{1+(10^{-\theta})^{\frac{-x}{z}+1}} = \frac{1}{1+(10^{-\theta})^{\frac{-\lambda x}{\lambda z}+1}}, \quad \forall\ \lambda \in \Re$$

$\mu(\cdot)$ is homogeneous of degree cero in incomes. ■

Proposition 2. Given θ different from zero, if the poverty line is z_0 then $\left|\mu(z_0 \pm d) - \mu(z_0)\right| = k_0$, $d > 0$, and if z_1 is an alternative poverty line then $\left|\mu(z_1 \pm d) - \mu(z_1)\right| = k_1$. Therefore, if $z_1 > z_0 \Rightarrow k_0 \geq k_1$. In particular, if $0 < \mu(z_0 \pm d) < 1$ then $k_0 > k_1$. Thus $\mu(\cdot)$ satisfies what we called the poverty line level effect.

Proof: Given that $\mu(\cdot)$ is invariant to scale and since we pass from z_0 to z_1 while the incomes do not change in the same proportion, then the slope of the *membership function* gets less steeper (see figure 7). ■

This property is very important since that given an income distribution it means that if we specify a poverty line the *membership function* will assign a certain weight to a certain change in a given income at a specified distance *d* from the poverty line. While if we specify a higher poverty line, it will be less critical a change in income at the same distance *d* from it, so the *membership function* is going to assign less weight in the latter case than in the former. This is true whenever the change takes place in that part of the *membership function*'s domain where it takes values between zero and one, but nor zero nor one.

3.3 Poverty Index Defined by α-cut

The first type of index proposed is:

$$FP1(\theta, \alpha, z) = \frac{|P_\alpha|}{N} \qquad (11)$$

where: P_α is an α-cut of the fuzzy set *P*, $|\cdot|$ is the cardinality operator and *N* is the number of agents in the population or in other terms $N = |X|$.

The poverty measure defined by *FP1* depends on three parameters α, θ and *z*. The parameter *z* is defined by the country-specific food basket and is therefore given. But in the case of α and θ their values have to be specified by the analyst. For α we propose three values that capture three different aspects of poverty. One of this values is $\alpha = 0.9$. In this case we can measure that proportion of agents that are with no doubt poor even considering that *P* is fuzzy, in other words, we are capturing those agents that are far below the poverty line. Another case is when $\alpha = 0.3$, for which the measure can be compared with the *headcount ratio* because it captures the same proportion of agents plus the proportion of agents with an income a little above the poverty line. Therefore this measure (compared to the *headcount ratio*) gives us an idea of the percentage of the total population that has an income very similar to the poverty line. And the third is $\alpha = 0.1$ that captures the quantity of agents that have a positive membership degree in poverty.

In the case of θ the value assigned would basically depend on the degree of fuzziness that the analyst wants to consider.

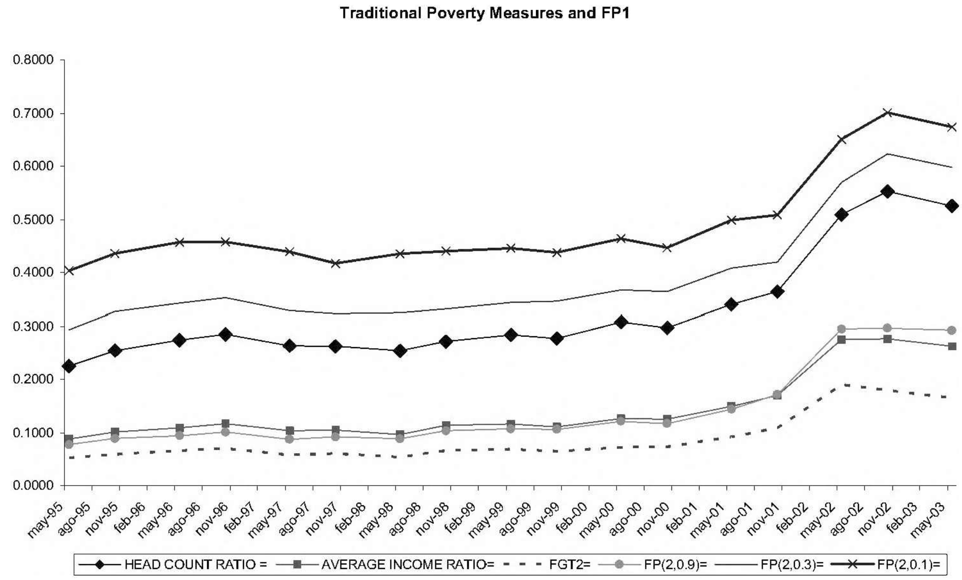
To illustrate *FP1* we have compute it using the EPH covering the period 1995-2003 surveyed in Gran Buenos Aires, Argentina. In addition, we have computed the *headcount ratio,* the *average income ratio* and FGT_2. In the process of computing all these indices we use the adjustments for *adult equivalents.* In table 1 we present the results of *FP1* for the period may 2003. And in figure 5 we present the evolution of all the indices for the entire period of analysis considering the parameters $\alpha = 0.1, 0.3, 0.9$ and $\theta = 2$.

TABLE 1

HEAD COUNT RATIO =	0.5253
AVERAGE INCOME RATIO=	0.2625
FGT2=	0.1653

	+ CRISP			+ FUZZY
	theta=500	theta=15	theta=5	theta=2
Alfa=0.9		0.4990	0.4380	0.2927
Alfa=0.3		0.5385	0.5604	0.5987
Alfa=0.1	0.5253	0.5545	0.6024	0.6743

FIGURE 5



The table 1 shows that when we consider poverty as a crisp set the *FP1* is equal to the *headcount ratio*. The most interesting part of the table is where we specify θ equal to 15, 5 and 2. In this manner, when we choose α equal to 0.1 we can see that the measure increases when θ decreases, reaching a value of 0.6743 which means that the 67.43% of the population has a certain association to poverty. This is not inconsistent since this measure is taking in account incomes that are below the $400 (see figure 4). Thus it provides information about how many people has an income similar to the poverty line and in the Argentinean case this information should be accounted since its significant magnitude (see Figure 5). On the other hand, the row of α equal to 0.3 shows how many people have an income below the poverty line plus the people that have an income scarcely above it. We can see that the quantity of people that have an income similar to the poverty line is very important and this information is lost in traditional poverty measures (see also figure 5). Finally, when we consider an α equal to 0.9 we are measuring how deep poverty is, this measure is very similar to the average income gap (we can see this in figure 5).

In resume, we can use this measure as a sensitive analysis for the *headcount ratio* because it provides information about how the latter could change if we move the poverty line in a few monetary units.

3.4 Poverty Index Defined Directly by the Membership Function

A second class of poverty index proposed is:

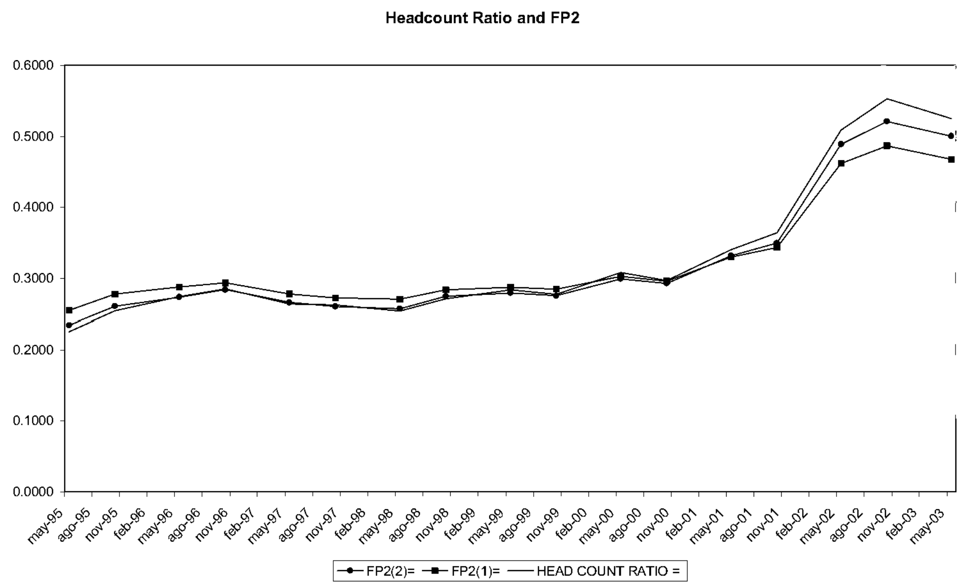
$$FP2(\theta, z) = \frac{\sum_{i=1}^{N} \mu(x_i)}{N} \tag{12}$$

The *FP2* index measures the expected membership grade in poverty in a given society. This measure depends, given other things, on the degree of fuzziness considered by the analyst.

In a subsequent section we show that this index has some advantages over *FP1*, however *FP1* is more flexible and potentially informative due to the possibility of changing α.

To illustrate *FP2* we have compute it using the same household survey and the same adjustments for *adult equivalents* used in the previous section. Figure 6 presents the evolution of the *headcount ratio* and the *FP2* for the entire period of analysis considering $\theta = 2,1$.

FIGURE 6



We can see in figure 6 that the *FP2* generates results that are very similar to the *headcount ratio,* but in some periods the former is above and in other periods it is below the latter. This is because the *FP2* satisfies the monotonicity axiom (we will show this in the next section) so we can argue that when FP2 is above the headcount ratio this is because the individuals are poorer. However, this proposition is false since in the next section we are going to show that the *FP2* violates the transfer axiom so the conclusions that we can draw from this relation turn ambiguous.

Therefore, comparing the performance of the two proposed measures the *FP1* provides more useful information than the *FP2.* Because it gives us additional information than the traditional measures used in the poverty line literature. While the *FP2* generates results that are very similar to the *headcount ratio* and in addition it introduces some ambiguities in the analysis.

3.5 About the Degree of Fuzziness

The issue of specifying the degree of fuzziness is in some form open to the analyst point of view. Nevertheless, we found some values that are very useful to study the Argentinean case and these could be helpful in the poverty analysis of another country. These values are: 2, 5 and 15 for the *FP1*, and 2 for the *FP2*.

But given that the *membership function* is invariant to scale the width of the range of incomes that are at the fuzzy zone depends on the level of the poverty line, figure 7 illustrates this idea. The first set of functions are specified with *z*=100 and the second set with *z*=200. We can see that given a degree of fuzziness the width of the range of incomes at the fuzzy zone duplicates if we duplicate the poverty line. We can see the same idea comparing figure 8 with figure 7, the former shows a function with z=1000 and $\theta = 2$, we can see that this in appearance is the same function showed in figure 7 for z=100 and $\theta = 2$.

The intuition given in this section helps us to choose which degree of fuzziness is better in a certain situation.

FIGURE 7

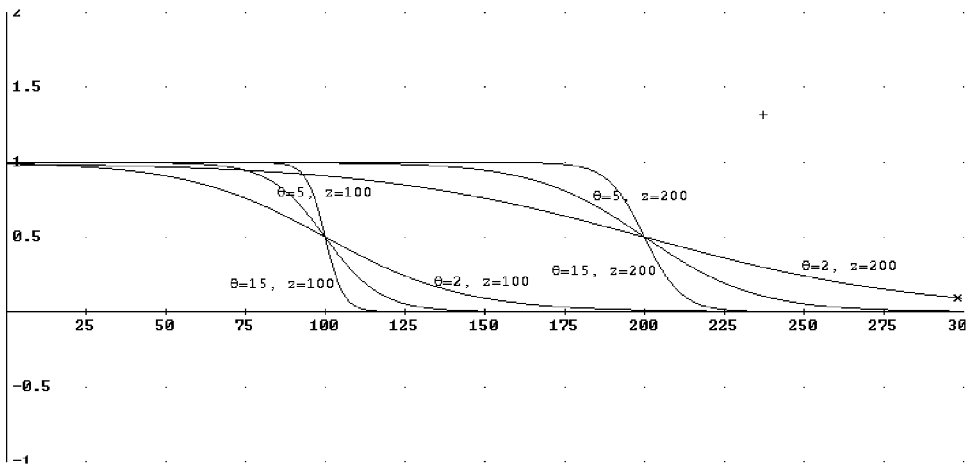
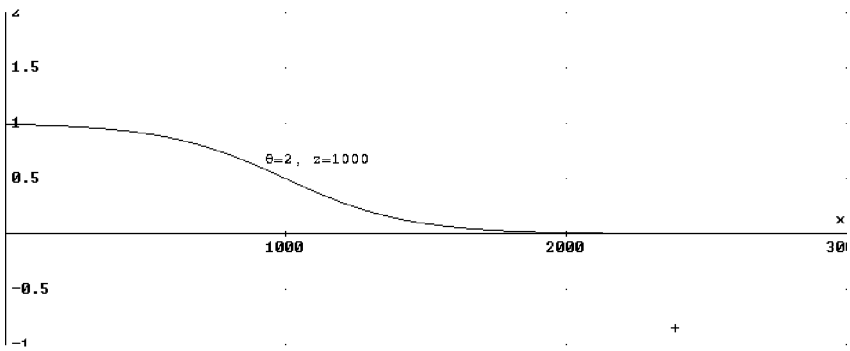


FIGURE 8



3.6 FP1, FP2 and the Satisfaction of Axioms

Since *FP1* and *FP2* are based on the *membership function* they inherit many of the properties of that function.

Proposition 3. *FP1* and *FP2* are invariant to scale and satisfy the poverty line level effect axiom.
Proof: Because *FP1* and *FP2* are derived from the *membership function.* ■

Proposition 4. *FP1* and *FP2* satisfy the subgroup monotonicity axiom.
Proof: By analogy with Foster et. al. (1984). ■

Proposition 5. *FP1* violates the monotonicity axiom.
Proof: By analogy with the *headcount ratio*. ■

Proposition 6. *FP2* satisfies the monotonicity axiom when θ >0.
Proof: Since $\mu(\cdot)$ is monotonic when θ >0. ■

Proposition 7. *FP1* and *FP2* violate the transfer axiom.
Proof: The case of *FP1* is similar to the case of the *headcount ratio* so by analogy with the latter it can be shown that the *FP1* violates the transfer axiom.
In the case of *FP2,* if a transfer t >0 of income takes place in the concave part of the *membership function* (when θ >0) from a poor individual *i* with income x_i to a individual with income x_i+*d* (*d*>0) then *FP2* falls. ■

3.7 Head Count Ratio as a particular case of a Fuzzy Index

Proposition 8. When $\theta \to \infty$ the *FP1* and *FP2* are equivalent to the *headcount ratio.*

In other words the *headcount ratio* is a particular case of the fuzzy measures *FP1* and *FP2* if we consider that poverty set to be crisp.

This property is very useful in the sense that FP1 and *FP2* satisfy the same axioms as the *headcount ratio*, but the usefulness comes from the fact that the *FP2* in addition satisfies the monotonicity axiom. Therefore, the failure to comply with the monotonicity axiom by the *headcount ratio* can be explained by the lack of enough fuzziness in the characterization of the poverty set.

3.8 Some Remarks from an Application of Fuzzy Set Theory to the Poverty Measurement

Let us define a measure based on fuzzy sets called *Well Behaved Fuzzy Poverty Measure* (W*BFPM*) which satisfies the following axioms:

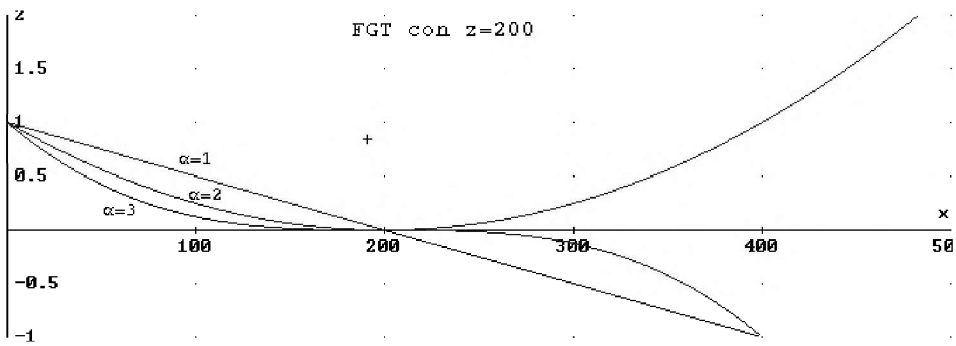
- Invariant to Scale.
- Monotonicity axiom.
- Poverty line level effect axiom.
- *BFPM* is equivalent to the *headcount ratio* if we consider the poverty set to be crisp.

Proposition 9. In a generic case, we can not postulate a *WBFPM* that satisfies the transfer axiom.
Proof: The transfer axiom is associated with a function that is strictly convex (see Foster et. al (1984)). Since no strictly convex *membership function* would satisfy the axioms satisfied by the *WBFPM*, we must drop the transfer axiom. ■

Figure 9 shows the *FGT* with z=200 and α equal to 1, 2 and 3. As we mentioned in a previous section this index satisfies the transfer axiom when α is greater than one. In this figure we can see clear why the transfer axiom is associated with a strictly convex function.

In addition to the result of proposition 9, we can say that in a generic case any measure based on fuzzy sets that wants to improve the poverty line measures and that has an economic sense will not satisfy the transfer axiom because it will not be possible to establish a *membership function* that is strictly convex.

FIGURE 9



4 Conclusions

In this work we have developed an extension of the *headcount ratio* index by means two new indexes based on the *membership function* of the poverty fuzzy set. The *FP2* measure allows the *headcount ratio* to satisfy the monoticity axiom. Moreover, the only axiom that this index does not satisfy is the transfer axiom.

On the other hand, the *FP1* based on α-cuts provides usefulness information that allows us to characterize poverty in a more comprehensive way.

The proposed indexes show desirable properties and improve on earlier works of poverty measurement using fuzzy set theory. The shape of the proposed *characteristic function* eliminates the arbitrariness of fixing bounds on the poverty set and introduces a fuzziness parameter.

Finally, we showed that in a generic case any measure based on fuzzy sets that wants to improve the poverty line measures must drop the transfer axiom.

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