Travelling Source Estimation using Spatio-temporal Data

Oscar Norberto Bria
onb@info.unlp.edu.ar

LIDI. Laboratorio de Investigacin y Desarrollo en Informtica.
Facultad de Informtica, UNLP.
50 y 115. 1er Piso. La Plata

Abstract

The estimation of travelling source parameters takes a considerable importance for many areas of sensor space-array processing, e.g., radar, underwater acoustic, non-invasive electro-medicine.
State-space models are a well suited framework for solving that dynamic estimation problem and they are in the core of our studies.
The parameter estimation problem is solved by analyzing spatio-temporal data, in applications where a relative large amount of noisy data is available.
Part of our project is to analyze the conjunction of state-space models and spatio-temporal data techniques for the estimation of the parameters of a travelling source. The technique will be used for the estimation of travelling brain sources from EEG/MEG measures.

State-Space Models for a Travelling Source

State-space models are two related equation for modeling two related process. One equation is a recursion describing the state transitions or travelling process. The other equation despicts the functional relation between the measurement process or observations and the states.

State-space models are easy to analyze when the two equations are linear because they lead, under certain noise assumptions, to the well known Kalman filter. When nonlinearities arise and can not be avoided, still the Kalman approach can be used with some modifications.

Models for Propagation

A travelling source is a source with variant spatial characteristics. We postulate that the motion of the travelling source is governed by a linear dynamic multidimensional system driving by non-correlated zero-mean Gaussian noise.

$$\boldsymbol{x}_{t+1} = \mathcal{F}_t\, \boldsymbol{x}_t + \boldsymbol{w}_t \tag{1}$$

where $\boldsymbol{x}_t$ is the unknown vector parameter of the source at time t, $\mathcal{F}_t$ is a known matrix describing the dynamics of the propagation, and $\boldsymbol{w}_t$ is characterized by a known symmetric covariance matrix $\Sigma_{\boldsymbol{w}_t}$. Equation 1 defines a *linear propagation model*.

Now, suppose that $\mathcal{F}_t$ depends upon an unknown vector of parameter or weights $\boldsymbol{s}_t$. We can compose a unique vector of unknown parameters $\boldsymbol{\vartheta}_t = [\boldsymbol{x}_t\ \boldsymbol{s}_t]$, and after grouping and extending with zeros, we obtain, in general, the following *nonlinear propagation model*.

$$\boldsymbol{\vartheta}_{t+1} \;=\; \Phi(\boldsymbol{\vartheta}_t) + \mathring{\boldsymbol{w}}_t \tag{2}$$

Above, to solve the problem of an unknown dynamics, the state vector is augmented by addition of the unknown parameters. This leads, in general, to the nonlinear dynamics equation (2) due to the occurrence of products between parameters and states [4]. But if we assume *polynomial dynamics* we can avoid the nonlinearity.

A particular case of the nonlinear propagation model is when $\boldsymbol{s}_t$ is modelled using a parametric recursion. In such a case we have the following digression

$$\boldsymbol{x}_{t+1} \;=\; \mathcal{F}(\boldsymbol{s}_t)\, \boldsymbol{x}_t + \boldsymbol{w}_t \tag{3}$$

$$\boldsymbol{s}_{t+1} \;=\; B\boldsymbol{s}_t + \boldsymbol{v}_t \tag{4}$$

with symmetric covariance matrix $\Sigma_{\boldsymbol{v}_t}$ for noise $\boldsymbol{v}_t$. Another particular case is when $\boldsymbol{s}_t$ is a constant, i.e., $\boldsymbol{s}_{t+1} = \boldsymbol{s}_t = \boldsymbol{s}$.

Models for Observations

We have just indirect access to the source parameters $\boldsymbol{x}_t$ and the travelling parameters $\boldsymbol{\vartheta}_t$ through measures coming from a postulated observation process.

In most cases, the following nonlinear equation results for the observations,

$$\boldsymbol{z}_t = m(\boldsymbol{x}_t) + \boldsymbol{\imath}_t \tag{5}$$

where $\boldsymbol{x}_t$ are the parameters of the source observed at time t, $m(\cdot)$ is the nonlinear relation between those parameters and the array observations $\boldsymbol{z}_t$, and $\boldsymbol{\imath}_t$ represent the observation and modelling noise which is assumed Gaussian, non-correlated and zero-mean. Note that equation (5) defines the so called *nonlinear observation model*.

From observation model equation (5) we can obtain an a priori estimate of $\boldsymbol{y}_t$ by applying any inverse technique for a single snapshot, e.g., single dipole analysis.

In the following, $\boldsymbol{y}_t$ is any estimate of $\boldsymbol{x}_t$ when only the information available at time t has been used in the estimation. Then a *linear observation model* is given by,

$$\boldsymbol{y}_t \quad = \quad \boldsymbol{x}_t + \boldsymbol{e}_t \tag{6}$$

with symmetric covariance matrix $\Sigma_{\boldsymbol{e}_t}$ for the estimation error $\boldsymbol{e}_t$.

State-Space Models

Our goal is either the estimation of the source position parameters $\boldsymbol{x}_t$, or the join estimation of both, the source parameters and the travelling model parameters, $\boldsymbol{\vartheta}_t = [\boldsymbol{x}_t \ \boldsymbol{s}_t]$. In general [1], that distinction is reflected in the nonlinearity or linearity of the propagation model.

We can estimate directly from the multiple snapshot process or we can re-estimate from the a priori computed single snapshot estimates. That distinction is reflected in the linearity or nonlinearity in the observation model.

Now we can combine any observation model with any dynamic model to build four state-space models with different complexity. For any one of them we can apply several Kalman-like techniques to solve the tracking/identification problems that arise [1], [2] [3], [4], [5], [6], [7].

Sumarizing, the state-space models we are analyzing are:

$$\text{model I} \qquad \begin{aligned} \boldsymbol{x}_{t+1} &= \mathcal{F}_t \, \boldsymbol{x}_t + \boldsymbol{w}_t \\ \boldsymbol{y}_t &= \boldsymbol{x}_t + \boldsymbol{e}_t \end{aligned}$$

$$\text{model II} \qquad \begin{aligned} \boldsymbol{x}_{t+1} &= \mathcal{F}_t \, \boldsymbol{x}_t + \boldsymbol{w}_t \\ \boldsymbol{z}_t &= m(\boldsymbol{x}_t) + \boldsymbol{\imath}_t \end{aligned}$$

$$\text{model III} \qquad \begin{aligned} \boldsymbol{\vartheta}_{t+1} &= \Phi(\boldsymbol{\vartheta}_t) + \mathring{\boldsymbol{w}}_t \\ \boldsymbol{y}_t &= \boldsymbol{x}_t + \boldsymbol{e}_t \end{aligned}$$

$$\text{model IV} \qquad \begin{aligned} \boldsymbol{\vartheta}_{t+1} &= \Phi(\boldsymbol{\vartheta}_t) + \mathring{\boldsymbol{w}}_t \\ \boldsymbol{z}_t &= m(\boldsymbol{x}_t) + \boldsymbol{\imath}_t \end{aligned}$$

[1] Recall that polynomial dynamics is an exception.

Estimation from Spatio-Temporal Data

Classical array processing rely upon short time analysis. Whatever the considered processing, they try to optimize a a spatial contrast at a given instant [9]. The integration time for the estimation of fixed parameters is limited by the basic non-stationarity of the received signal due primarily to the travelling of the source.

When the temporal resolution is relatively large compared to the travelling characteristics of the source, then it is possible to consider relatively large integration times for detecting weak signals. The integration time defines the interval of the spatio-temporal data during which some parameters are consider fixed. That interval corresponds to a single snapshot in the state-space models. A spatio-temporal model describe those parameters of the source model that remain fixed and those which are unconstrained along the integration time.

Well known algorithms for the analysis of spatio-temporal data are the MUSIC-like scanning function [8]. Reconsider equation 5 dropping the temporal dependency. Assume that $m(\cdot)$ is unknown since we do not yet know $\boldsymbol{x}$. A data record in a single trial is composed of N time samples $Z = [\boldsymbol{z}_1, \cdots, \boldsymbol{z}_N]$. Define:

$$\bar{Z} = \sum_{k=1}^{K} Z_k / K \tag{7}$$

$$\hat{R} = \sum_{k=1}^{K} Y_k Y_k^T / NK \tag{8}$$

and let V_{nr} be a matrix whose columns are the (normalized) eigenvectors corresponding to the largest nr eigenvalues of $(\hat{R}^{-1/2}\bar{Y}\ \bar{Y}^T\hat{R}^{-1/2})/N$. It can be shown that $I_m - V_{nr}v_{nr}^T$ is the projection matrix onto the space orthogonal to the column space of $\hat{R}^{-1/2}\hat{m}$, where $\hat{m}$ is a ML estimate of m. Then a scaning function can be easily constructed as

$$l(\theta) = \frac{1}{\lambda_{MIN}\left(m(\boldsymbol{x})^T\hat{R}^{-1/2}[I - V_{nr}V_{nr}^T]\hat{R}^{-1/2}, m(\boldsymbol{x})^T\hat{R}^{-1}m(\boldsymbol{x})\right)} \tag{9}$$

where $\lambda_{MIN}(\cdot,\cdot)$ denotes the smallest generalized eigenvalues of the matrices in the parenthesis and $m(\boldsymbol{x})$ is a single source array response matrix.

When the signal is weak whereas the temporal resolution is relatively large it is on certain occasions likely to use MUSIC for single snapshot parameter pre-estimation by integrating over a sliding temporal window.

After pre-estimation it is feasible to smooth the estimation using Kalman filtering or smoothing techniques by incorporating dynamic information and/or constraints. Observe that the state-space model that applies is model I or model III.

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