PRE-PLAY RESEARCH IN A MODEL OF BANK RUNS

J. DANIEL AROMÍ

RESUMEN

Se estudia un modelo de corrida bancaria en el que los depositantes no llegaron a un consenso sobre la acción a elegir pero pueden informarse acerca de las intenciones de los otros jugadores. La propiedad de equilibrio indeterminado reaparece en el juego con adquisición de información. En el escenario bajo análisis, el equilibrio con niveles positivos de adquisición de información resulta en mayores probabilidades de quiebra del banco. La correlación en las señales aumenta el espacio de parámetros para el cual existen equilibrios con niveles positivos de adquisición de información y mayor probabilidad de quiebra del banco.

Clasificación JEL: C72, G21.

Palabras Clave: Corridas bancarias, Coordinación.

ABSTRACT

We consider an extension of a bank run game assuming that depositors do not agree on the action to be taken but can perform research in order to learn about others' initial intentions. Equilibrium indeterminacy, a characteristic of bank run games, re-emerges in the game with information acquisition. In the configuration under analysis, the equilibrium with positive levels of information acquisition results in higher probability of bankruptcy. Additionally, correlated signals enlarge the set of parameter values such that there exist equilibria with positive levels of information acquisition and higher probability of bankruptcy.

JEL Classification: C72, G21.

Keywords: Bank runs, Coordination.

PRE-PLAY RESEARCH IN A MODEL OB BANK RUNS

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I. Introduction

This work studies a bank run game in which players are uncertain about the beliefs and tentative actions of other players. This is a reasonable assumption in ambiguous and rare circumstances such as scenarios of potential bank runs. In these cases, multiple equilibria can emerge and different players might arrive to reasonable but different interpretations of the strategic interaction in which they take part.

Given strategic uncertainty, players might engage in information acquisition activities. We contemplate this possibility assuming players can collect information about other players' likely actions. More specifically, before play, they can receive a signal about the tentative play of others'.

We are interested in establishing the effect of information acquisition on the equilibrium indeterminacy characteristic of bank run games. More specifically, we assume an information structure such play is well determined and evaluate whether an information acquisition stage implies multiple equilibria. Additionally, we are interested in describing the impact of information acquisition on the likelihood of successful runs.

The game exhibits strategic complementarities in research activities. Thus, in general, multiplicity of equilibria cannot be ruled out in the extended model. We identify conditions under which equilibrium indeterminacy re-emerges in the game with information acquisition. With respect to the second issue, for the configuration under analysis, the equilibrium with positive levels of information acquisition results in higher probability of bankruptcy. In addition, we show that correlated signals enlarge the set of parameter values such that there exist equilibria with positive levels of information acquisition and higher probability of bankruptcy.

These results have implications for the convenience of different forms of information transmission in circumstances of strategic uncertainty. They suggest that communication can be harmful and correlated messages can be

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worse than independent messages. Additionally, these results suggest that multiple equilibria are a pervasive condition. In our setting, we specify beliefs such that optimal play for each players is determined, but indeterminacy reappears once the ability to collect information is contemplated. Multiple equilibria imply that the structure of the strategic interaction does not lead to an unambiguous prediction of actions selected by players. One interpretation of this condition is that the theory needs to be complemented by an understanding of how players learn to interpret their environment.

We believe that the insights presented in this analysis are also relevant in other economic interactions beyond bank runs. In particular, in circumstances where there exist strategic complementarities and previous play is not available or is not expected to be a reliable guide for future play. A heterogeneous but incomplete list of examples would include: economic development, technology adoption, geographic location, debt crises, currency attacks, arbitrage in financial markets, business cycles, bargaining and pro-social behavior.[1] For this relevant class of settings, the usual argument of coordination as a result of a dynamic process of convergence does not apply. As a result, there is value in exploring alternative representations of the process that shapes behavior.

Our contribution can be understood as one evaluation of how play can emerge in a context of high strategic uncertainty. In this representation we do not assume common knowledge of rivals' play but assume a weaker assumption regarding the structure of information. We assume that beliefs about likely play have a common prior, difference of opinion are only the result of incoming signals. The weaker, but still restrictive, assumption of common prior beliefs can be understood as approximating situation in which, at a point in time, players share a common history and all players have built a consensus about the lessons of history and about the likely, but uncertain, reaction to news. Our results indicate that multiple equilibria might emerge. That is, even when tentative decisions are completely specified, strategic uncertainty reemerges when cognitive decisions are allowed.

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[1] For example see: Murphy et al. (1989) for economic development, Arthur (1999) for technology adoption, Avery et al. (1994) for bargaining, Abreu et al. (2002) for arbitrage, Cooper and John (1988) for business cycles, Rabin (1993) for pro-social behavior, Krugman (1991) for economics and geography, Diamond Dibvig (1983) for bank runs, Morris et al. (2004) for financial liquidity, Obstfeld (1996) for currency crises, Calvo (1988) for debt crises.

The following section presents a brief revision of related literature. Section 3 presents the extended game. The next section analyzes the strategic properties of the game. The final section presents some concluding remarks.

II. Related literature

This contribution is linked to a significant body of literature that studies events of bank runs. The seminal contribution of Diamond et al. (1983) shows that bank run events can be associated to circumstances of multiple equilibria. There exist parameter regions in which there exist an efficient equilibrium and an inefficient equilibrium. In the efficient equilibria deposits are renewed by depositors that have no liquidity needs. This behavior is in equilibrium if fundamentals are strong enough so that banks are able to comply with their contractual obligations. In the inefficient equilibria, all depositors withdraw their deposits, productive investment projects are liquidated in an inefficient manner and, in this way, the bank goes bankrupt. Withdrawing the deposit is a best response as long as the inefficient liquidation of investment projects implies sufficiently low payoff for a depositor that renews its deposit.

The literature has been expanded in different directions. Postlewaite et al. (1987) show that asymmetric information can result in a unique equilibrium. Rochet et al. (2004) apply the global games framework[2] to obtain a unique equilibrium and in this way establish connections between game parameters and the probability of a bank run. They focus on the value of a lender of last resort. Similarly, Golstein et al. (2005) apply global game techniques to obtain unique equilibria and analyze the desirability of demand deposit contracts. Green et al. (2000) and Peck et al. (2003) focus on the use of more complex contractual agreements and its capacity to eliminate inefficient equilibria.

Nikitin et al. (2008) and Hasman et al. (2008) study information acquisition in bank run scenarios. In these studies the information to be acquired deals with the evaluation of projects that have been funded by banks. In contrast to these contributions, in our framework we focus on imperfect information regarding other players' beliefs.

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[2] We provide a short description of the global games framework in the more general discussion of multiple equilibria in the next paragraphs.

An experimental analysis on bank runs has been developed by Garratt et al. (2009). The author assesses how the probability of a successful run depends on uncertainty regarding the fraction of depositor with liquidity needs and the number of opportunities to withdraw their deposits. Both factors are found to increase the probability of successful runs.

More broadly, our analysis can also be understood as part of a series of more general analyses focused on the prediction of play in games with multiple equilibria. The presence of multiple equilibria indicates that the theory used to explain or predict behavior is incomplete in the sense of not being able to identify a unique predicted outcome. Naturally, this scenario stimulated analyses that try to enrich the theory in order to eliminate the observed incompleteness.

One simple account postulates that there exist "sunspots" through which agents coordinate towards specific profiles of consistent plans. This explanation is not satisfactory since it amounts to ignoring or assuming away the strategic uncertainty that characterizes this environment. Alternatively, it has been argued that equilibrium is selected according to evaluations of each equilibrium associated levels of risk and payoff.[3] While considerations of average payoffs and risk are likely to influence behavior, these arguments on equilibrium selection need to be accompanied by a description of the process through which agents agree on the valuation of different equilibria.

In a different approach, coordination toward equilibrium has been explained as a result of a dynamic process in which agents learn about other's likely play as they interact.[4] These analyses typically assume a stable environment that allows for convergence to equilibrium. As indicated above, our description is focused on a different type of scenario. We focus on the case in which history is non-existent or not very informative. Nevertheless, we envision that our insights could be informative in a dynamic analysis of action selection in stationary and non-stationary environments.[5]

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[3] See Harsanyi and Selten (1988) for a comprehensive presentation.

[4] See Fudenberg and Levine (1998).

[5] Explanations based on the concept of a focal point can be viewed as incorporating both elements of sunspots and dynamic learning. Focal points permit predicting actions as a function of the shared opinions about the prominence of an option and independently of its payoffs. In this approach, coordination emerges as a result of reasoning about these shared opinions that are formed most likely through learning in repeated interactions.

Another strategy for tackling strategic uncertainty involves considering incomplete information. In this type of settings, players observe their signal and, given a distribution of other players' information which is common knowledge, select best responses to the contingent plans of rivals which are predicted to be consistent. Heterogeneous private signals about payoffs can lead to uniqueness in the class of consistent plans in which play is conditional on a private signal (for the seminal contribution see Carlsson and Van Damme (1993) or Morris and Shin (1998) for a high impact contribution).[6] Uniqueness results are based on the strong assumption of common knowledge in prior beliefs. In addition, under asymmetric information, uniqueness is a result of anchoring play through contingent plans for scenarios considered very unlikely once each player conditions on the private signal.

This refinement through incomplete information has been scrutinized in terms of the robustness of its predictions. For example Costain (2007) and Angeletos et al. (2006) show that introducing dynamic elements can result in the re-emergence of multiple equilibria despite the presence of asymmetric information. We do not explore this type of dynamic extensions of a game with multiple equilibria but we believe that interesting insights could result from the joint consideration of pre-play research stages and sequential choices of actions.

The contributions focusing on communication in games with complete information about payoffs share characteristics with our contribution.[7] These contributions typically consider situations in which two players have different preferences about the preferred equilibrium. Situations in which players communicate before playing a game with Pareto ranked equilibria or games of voluntary contributions have also been considered.[8] In an enlarged game, messages are sent about intended play in order to coordinate play. Similarly, our formulation considers an enlarged game in which players learn about the intention of play of others. But in our formulation we have many players, there is no sender and learning takes the form of a noisy signal of rivals' play that cannot be manipulated by a sender. In other words, our pre-game exchange of

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[6] As indicated above, Postlewaite et al. (1987), Rochet et al. (2004) and Goldstein et al. (2005) are examples of models that assume asymmetric information in bank run scenarios.

[7] Farrell (1987), Rabin (1994) considers communication as a coordination device when there is complete information about payoffs.

[8] For a recent experimental contribution see Bochet and Putterman (2009).

information is unilateral, each player decides whether to extract a signal informative of many players' intentions.

The numerous and diverse contributions that tackle the challenge posed by multiple equilibria are suggestive of the limitations of an agenda that intends to impose a single criteria. In other words, in interactions with multiple consistent plans, there are multiple potential refinements and, as a result, strategic uncertainty can be rephrased as "equilibrium refinement uncertainty". Having these considerations in mind, next, we analyze a setting in which there is no common knowledge of rivals play but information structure satisfy the milder but strong assumption of common priors.

III. A model of bank runs with pre-play research

Before introducing the information acquisition stage, we present a simple model of bank runs. There is a continuum of measure N of depositors that simultaneously choose whether to renew or withdraw their funds. The amount of each deposit is equal to 1. If the amount of withdrawn funds is below L, then the depositors that renewed their deposits earn a payoff equal to $1 + d$ and those that withdrew their funds earn a return equal to 1. If the amount of withdrawn funds exceeds L then the bank goes bankrupt. In this case, the depositors that withdrew their funds receive a payoff equal to $z < 1$ and the payoffs for depositors that did not withdraw is 0. This is a simplified version that does not include impatient depositors (those that need to withdraw independently of the likelihood of bankruptcy). In addition, we assume a very abrupt change in average payoffs implicitly associated with a large negative impact of bankruptcy on the value of the bank net assets.[9] Finally, we are not describing the productive use of the funds that are borrowed by the bank. These simplifications are made to secure a more tractable analysis; qualitatively similar results would hold for versions that are closer to the traditional model.

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[9] Additionally, this payoff structure is a good approximation of what would be observed if the players in this game were junior debt holders. In this case, the cost of the bankruptcy process would lead to a violent negative adjustment in their payoffs but, conditioned on bankruptcy, their payoffs would not change significantly with the number of withdrawals.

An action profile a assigns an action $a_i \in \{r, w\}$ to each player. For a player i, the payoff function can be expressed as a function of the action selected by player i, a_i, and the quantity of players selecting action w, a_w:

$$
u_i(a_i, a_w) = \begin{cases} 1 + d & if\ a_i = r\ and\ a_w \leq L \\ 0 & if\ a_i = r\ and\ a_w > L \\ 1 & if\ a_i = w\ and\ a_w \leq L \\ z & if\ a_i = w\ and\ a_w > L \end{cases}
$$

It is easy to establish that, as long as L is less than N, there exist two Nash Equilibria involving pure strategies. In one equiilibria all players renew their deposits earning a payoff of $1 + d$ which is higher that the payoff of 1 associated to selecting the alternative pure strategy. In addition, there exists another equilibrium in which all depositors choose to withdraw their deposits, in this case their payoffs equals z which is higher than 0 the payoff associated to renewing the deposit. The first equilibrium is the payoff dominant equilibrium.

We now develop a variation of these game in which aggregate behavior is unknown and players can acquire information. The extension involves assuming multiple types endowed with tentative actions and beliefs regarding rivals' tentative play. In addition, there exists a pre-play stage where players can acquire a signal that is informative about the tentative play of other players. The initial profile of tentative play is given by a^0. Each type of player is endowed with subjective beliefs defined over a^0 given by a probability distribution function $F_i(a^0)$. We assume that each player tentative action and subjective beliefs are consistent in the sense that a_i^0 maximizes expected payoffs when the action profile is believed to satisfy $F_i(a^0)$. That is, as long as beliefs imply that the optimal action is unique, a type is completely determined by its beliefs. Taking the tentative play and beliefs associated to tentative play as given, we present a game in which agents can acquire information and revise its tentative play.

Research activities are developed simultaneously. In this pre-play stage, each player sets the level for an activity that increases its information about rival's tentative play. Let $b_i \in \{0,1\}$ represent the level of this research activity. In order to distinguish this activity from the action in the second stage, we will use the expression "research action" to refer to the choice of b_i.

If $b_i = 1$, player i will receive a message $m_i \in \{w, r\}$, a noisy signal about the tentative action profile a^0. The signal is w with probability equal to the fraction of players tentatively selecting w and the signal is r with probability equal to the fraction of players tentatively selecting r. We assume that no signal is received if there is no research. We will denote this event by $m_i = \emptyset$. Research activity is potentially costly and is associated to a negative impact on payoffs equal to $c \geq 0$. Let b be the profile of research action and a^1 be the "post-research" profile of withdrawal or renewal decisions. After the research stage, each player forms beliefs about action profile a^1 and chooses action $a_i^1 \in \{w, r\}$.

As a result, this version of the model is a game with asymmetric information where each agent has a type determined by its tentative action and the beliefs regarding others' tentative action. Each player makes two decisions. The first action is given by a level of research activity, $b_i \in \{0,1\}$. After making this decision and updating beliefs, the choice between renewing the deposit or withdrawing the funds is made, $a_i^1 \in \{w, r\}$. Let a_w^1 be the fraction of players that, under profile a^1, select w. Then, the payoff function of player i is given by: $U_i(a_i^1, a_w^1, b_i) = u_i(a_i^1, a_w^1) - cb_i$ where $u_i(.)$ is the payoff function of the original game.

The research activity described above is a very specific form of information acquisition. In general, information acquisition can involve many different practices including word of mouth communication[10], attending to mass media content[11] or analyzing statistical information. The specification in our model seems to be closer to the case of word of mouth communication in a case in which the sender does not act strategically and the matching between subjects is random. The cost c of research activities can be associated to the opportunity cost of the cognitive resources allocated to this activity.

As indicated earlier, we assume learning activities are simultaneous. In our simple representation, all agents observe a signal of the same profile of tentative play. A more realistic representation would allow for sequential, but still unobserved, learning activities in which agents observe signals corresponding to different profiles of tentative play as research leads to

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[10] For studies of word of mouth communication see for example Banerjee et al. (2004) and Cao (2011).

[11] See for example Veldkamp (2006) and Tetlock (2007) for studies of the impact of information spread through media outlets.

changes in intentions. In this setting, players choosing to perform learning activities would benefit from being the last to get a signal of the tentative profile. This is because agents are interested in what other players will do, there is no intrinsic value in learning about what others' think. Our simultaneity assumption avoids the difficulties that arise from these asymmetries. At the same time, numerical simulations indicate that the quantitative differences of introducing sequential learning are relatively low.[12]

Before proceeding to the analysis of strategic properties of the game we would like to make an observation on spillovers in this game. We find that the welfare impact of research on others depends on the tentatively selected action of the player performing this activity. The spillover is negative in the case of players that have tentatively chosen to renew the deposit. This is because the number of players that withdraw their deposits can only increase as players with tentative play r perform research activities that can change their tentatively selected action. This results in negative externalities as, for a fixed distribution of other players' actions, the probability of bankruptcy increases. On the other hand, through a similar reasoning, we can verify that research by agents that have tentatively selected w leads to positive externalities.

IV. Equilibrium analysis

In this subsection we analyze equilibria of this game under the assumption of common prior beliefs. That is, we allow for differences of opinion but we assume that players' differences of opinion are only explained by differences in the information perceived. In this way, we select a specific information structure and, given these setup, we perform equilibrium analysis of the game that includes a stage of information acquisition. In the following subsection we specify the initial information structure. Next, we analyze equilibrium properties of the game. The last subsection presents a modified version of the game in which signals are correlated.

Before developing the analysis, it is convenient to make some remarks regarding the solution concepts. More specifically, in this work we allow for inconsistent behavior regarding financial decisions but we solve for an

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[12] It is worth noting, in contrast to the case of sequential research considered in this paragraph, sequential renewal or withdrawal decisions would lead to important consequences in play (Costein, 2007).

equilibrium in which information acquisition decisions are consistent. First, the analysis can be understood as an exploration of the likely behavior that might emerge under heterogeneous levels of belief coordination. More specifically, actors might have problems predicting beliefs in specific scenarios but they might be able to coordinate beliefs regarding how optimism or pessimism leads to information acquisition. For example, this would be the case if financial decisions depend on complex perception problems that resolve ambiguity while information acquisition is coordinated though some simple focal public signal.

From a different perspective, the exercise can be understood as an exploration in which the existence of multiple equilibria in the information acquisition stage is evaluated. If multiple equilibria are found then, it is established that the context is ambiguous, the coordination of decisions might not be achieved and path dependence is quite relevant. These are important messages that emerge from an equilibrium analysis. While both perspectives are admissible, we would tend to focus on this second view.

IV.1. The basic bank run game and its initial information structure

The analysis below will be carried out assuming parameter values that satisfy certain conditions that are specified at this stage. We assume $3/5 < L < 4/5$, that is, the ability of the bank to withstand liquidity shocks is moderately high. We also impose conditions on the payoffs associated to each outcome. We assume $z/9 < d < 2z/3$, that is, it is established that the interest rate is not too large or too small compared to the recovery rate under bank failure. The structure of information described above is consistent given the structure of parameter described in this paragraph.

We assume that there exist two types of players, a pessimistic type p and an optimistic type o. The tentative action of a pessimistic type is "withdraw" and the tentative action of an optimistic type is "renew". Given the information structure described below, these tentative actions are shown to be best responses to subjective beliefs. Each type is also characterized by a probability distribution function F_i with domain $[0, N]$. This distribution F_i constitutes the subjective beliefs of type t_i with respect to the frequency of types, that is $F_i(x) = Prob(a_w \leq x)$. Nature selects with equal probability a frequency x from a set of feasible frequencies X. Given the common prior assumption, the

beliefs of each type must satisfy the condition that these beliefs are equal to the updated beliefs of an agent that knows the process through which nature selects frequencies and is informed about its type.

We will propose and work with one specification that satisfies this condition. Nature selects with equal probability among four possible frequencies or states: pessimistic (P), medium-pessimistic (MP), medium optimistic (MO) and optimistic (O). The proportion of pessimistic agents are 4/5, 3/5, 2/5 and 1/5 respectively. The common prior assumption is that each agent knows this process.

At this point, it is convenient to make some remarks about the structure of information. The different states selected by nature can be interpreted as aggregate variations in the level of confidence that is controlled by complex processes that each agent is unable to understand. The idea is that there are many details that can affect the way in which the ambiguity presented by a specific concrete scenario is resolved by the perceptual system. These resolutions determine the proportion of optimistic players and pessimistic players. On the other hand, it is assumed that the agents share beliefs regarding the likelihood of each state. This coincidence in prior beliefs can be considered to be the long run result of shared experience in, or shared knowledge about, different but similar scenarios.

Beliefs are updated through Bayes' rule after an agent is informed about its type. For example, after receiving a message indicating a pessimistic type, updated beliefs regarding the probability of state H satisfies:

$$\Pr_i(x = P | t_i = p) = \frac{\Pr_i(x = P)\,\Pr_i(t_i = p | x = P)}{\Pr_i(t_i = p)} = \frac{1/5}{1/2} = \frac{2}{5}$$

Similar calculations show that according to updated beliefs states P, MP, MO and O are assigned probabilities 2/5, 3/10, 1/5 and 1/10 respectively. For the case of an agent with an optimistic type the updated beliefs are such that states P, MP, MO and O are assigned probabilities 1/10, 1/5, 3/10 and 2/5 respectively.

Absent any innovation to information sets, a pessimistic type will find it optimal to withdraw the deposit if:

$$E[u_i(w, a_w) | t_i = p] > E[u_i(r, a_w) | t_i = p]$$

$\text{Pr}_i(x = P|t_i = p)\, z + [1 - \text{Pr}_i(x = P|t_i = p)] > [1 - \text{Pr}_i(x = P|t_i = p)](1 + d)$

$2z/5 + 3/5 > (1 + d)3/5$

$2z/3 > d$

Similarly, it can be shown that an optimistic agent will find that r is a best response as long as $d > z/9$. That is, the tentative action if each type is consistent with its beliefs. This set up can be understood as a form of correlated equilibrium in which there behavior and beliefs are coordinated through aggregate shocks.

As indicated below, the parameter restrictions imply a scenario in which fundamentals are relatively strong and agents are sufficiently optimistic on average so that, absent information acquisition, bankruptcy would occur with probability $1/4$, that is, only in state P. Additionally, the parameter configuration imposes intermediate values of d and z so that, without information acquisition, pessimistic and optimistic players select different actions.

The information structure rules out the strong assumption of common knowledge regarding behavior and imposes a milder, but still strong, form of information coordination. The lack of consensus is only due to the arrival of new signals that determine the type of each player. This scenario can be interpreted as a situation in which, up to a specific point in time, all players share a common history and all players have built a consensus about the lessons of history. In addition, all agents know that the consensus is broken by the arrival of information that results in different types endowed with different beliefs. In this way, the consensus is partially broken, but in an orderly fashion.

Given this information structure and the game payoffs, in the absence of information acquisition, the behavior in this game is completely specified and optimal given beliefs. A pessimistic type selects "withdraw" and an optimistic type selects "renew". These are the best responses given their correct information about the beliefs of each type and their imperfect information regarding the proportion of players of each type. Bankruptcy occurs only in state P.

Our analysis focuses on the changes associated to allowing players to acquire information regarding likely play. In particular, we ask whether the equilibrium indeterminacy reemerges as a consequence of introducing an

option to learn about others' information sets. Additionally we are interested in describing the impact of information acquisition on the outcomes of the game.

IV.2. Equilibrium analysis with information acquisition

Turning back to the game with a research stage, we would like to define an equilibrium. A strategy involves the selection of a research decision for each type of agent. Formally, a strategy is a contingent plan $s = \{\{b_i\}_{i=o,p}, \{a_{im}\}_{i=o,p\ m=\emptyset,w,r}\}$ where $b_i \epsilon \{0,1\}$ is the research decision made by an agent of type $i \epsilon \{o,p\}$ and $a_{im} \epsilon \{w,r\}$ is the decision made by an agent of type $i\epsilon\{o,p\}$ that receives message $m = \{\emptyset, w, r\}$. It is assumed that, conditioned on the state selected by Nature, the signal received by players acquiring information is independent across players. The analysis will focus on symmetric equilibrium, that is, one in which each agent with the same type or information set selects the same action.

Definition:
A symmetric equilibrium of the bank run game with information acquisition is given by a strategy s^* such that:

$$a_{im}^* = arg \max_{a \in \{w,r\}} E[u_i(a, a_w)|t_i, m, s^*] \ for \ t_i = p, o \ and \ m = \emptyset, w, r$$

$$b_i^* = arg \max_{b \in \{0,1\}} V(b; t_i, s^*) \ for \ t_i = p, o$$

Where a_w is the random fraction of players selecting w under s^* and $V(b; s^*, t_i) = E[E[u_i(a_{im}^*, a_w)|t_i, m, s^*]|b]$ is the expected payoff for a player with type t_i when its research decision is b and all other choices in decision nodes (own and others') are as prescribed by s^*.

Equilibrium involves the usual optimality conditions. In addition, the expectation operators indicate the presence of belief updating. The following lemma identifies the conditions under which there exist an equilibrium with no research activity.

Lemma 1:

There exists an equilibrium path involving no research activity by any type as long as $c \geq \max\left\{\frac{8}{25}d - \frac{2}{25}z, \frac{2}{25}z - \frac{8}{25}d\right\}$.

Proof:

If no research is carried out then, beliefs regarding other's beliefs are not modified with respect to the initial profile, hence optimal decision making requires that, in the equilibrium path, optimistic players select action r and pessimistic players select action w. Additionally, the best response of an optimistic (pessimistic) player that receives signal r (w) is given by r (w). This is simply because these messages only strengthen the initial inclination of each type. Hence, the candidate equilibrium strategy is given by $s^* = \left\{0, 0, w, r, a^*_{pr}, r, w, a^*_{ow}\right\}$. Note that two elements in the contingent plan are not determined yet, this is because the optimal action in each of those situations depends on parameter conditions that are analyzed below. The analysis will consider two cases.

If $\frac{d}{z} \geq \frac{1}{4}$ only players with type p would benefit from a deviation. This is because after receiving $m = w$ an optimistic agent believes that the probability of frequency P is 1/5 and, given these assessment, it is easy to verify that the optimal action is renewing the deposit if:

$$\frac{4}{5}(1 + d) \geq \frac{4}{5} + \frac{1}{5}z \Leftrightarrow \frac{d}{z} \geq \frac{1}{4}$$

Then, in this case, information has no value for an optimistic agent since it does not lead to changes in decision making. On the other hand, a pessimistic agent that perceives $m = r$ would assign a probability of 1/5 to the event of bankruptcy. Again, in this case, selecting r is optimal. The value of information for this type is positive since it results, ex ante, in better decision making. The value of information is equal to the difference between the expected payoff when information is acquired and the payoff with no research activity. With no research activity the payoff is equal to:

$$V(0; s^*, p) = (1 - \Pr(P|t_i = p)) + z\Pr(P|t_i = p)$$

$$V(0; s^*, p) = \frac{3}{5} + \frac{2}{5}z$$

With research activity, the payoff is a function not only of the frequency of different type of players but also of the message perceived and the associated action selected:

$$
\begin{aligned}
V(1; s^*, p) \quad &= \ \Pr(P|t_i = p)\,[\Pr(m = w|P)z] \\
&+ \Pr(MP|t_i = p)\,[\Pr(m = w|MP) \\
&+ (1 + d)\Pr(m = r|MP)] \\
&+ \Pr(M0|t_i = p)\,[\Pr(m = w|M0) + (1 + d)\Pr(m = r|M0)] \\
&+ \Pr(O|t_i = p)\,[\Pr(m = w|O) + \Pr(m = r|O)(1 + d)]
\end{aligned}
$$

$$
\begin{aligned}
V(1; s^*, p) \quad &= \frac{2}{5}\left[\frac{4}{5}z\right] + \frac{3}{10}\left[\frac{3}{5} + (1 + d)\frac{2}{5}\right] + \frac{1}{5}\left[\frac{2}{5} + \frac{3}{5}(1 + d)\right] \\
&+ \frac{1}{10}\left[\frac{1}{5} + \frac{4}{5}(1 + d)\right]
\end{aligned}
$$

$$V(1; s^*, p) \quad = \frac{8}{25}z + \frac{3}{5} + \frac{8}{25}d$$

The value of information is given by:

$$V(1; s^*, p) - V(0; s^*, p) = \frac{8}{25}d - \frac{2}{25}z$$

Then, for $\frac{d}{z} \geq \frac{1}{4}$, as long as c is larger then or equal to the expression above, setting $b = 0$ is an optimal action.

For the case $\frac{d}{z} < \frac{1}{4}$, a similar analysis indicates that the optimistic type is the only one that assigns a positive value to the signal. The value of the signal can be shown to equal:

$$V(1; s^*, o) - V(0; s^*, o) = \frac{2}{25}z - \frac{8}{25}d$$

Then, for the case $\frac{d}{z} < \frac{1}{4}$, as long as c is larger or equal than the expression above, setting $b = 0$ is an optimal action. □

This lemma indicates that, for any value for parameter c, there is an equilibrium without information acquisition as long as the parameters d and z are not too far away and the cost of information acquisition is sufficiently high. If this cost is too low, it will be the case that one type of player will find it optimal to deviate and acquire information. The cost parameter is compared against the value of information of the type of player that, as a function of payoff parameters, is the one that assigns the highest value to information.

It must be noted that the value of information is endogenous, that is, it depends on the decisions made by other agents. The way in which the value of information varies with information acquisition decisions is the key property that will determine the existence of multiple equilibria. The next lemma evaluates the conditions for the existence of symmetric equilibria with positive levels of research activity by optimistic agents.

Lemma 2:

There exists an equilibrium path involving research activity by optimistic types only as long as $\frac{1}{5}z \le d$ and $c \le \max\left\{\frac{1}{2}d - \frac{1}{10}z, \frac{1}{5}z - \frac{1}{5}d\right\}$.

Proof:

In the corresponding equilibrium path, pessimistic agents do not perform research activities and select action w. This is because their beliefs regarding the proportion of each type does not change and they anticipate some optimistic might change their mind. Additionally, for optimistic agents to perform costly research activities in equilibrium, it must be the case that the resulting information should be valuable. In other words, the optimal choice of action in the second stage must be w if the message is w and must equal r if the message is r. This implies changes in the frequencies of players choosing to withdraw their funds with respect to the original frequencies determined by the fraction of players of each type. In particular, under state MP, there are 2/5 of optimistic players receiving signal w. This implies that after research activities are performed, the fraction of players selecting w is equal to the

fraction of pessimistic agents plus the fraction of optimistic players times the probability of receiving message w. It is easy to check that the number is higher than 4/5. This implies that, under the postulated parameter values, bankruptcy will be observed under state MP. In states O, MO and P, the fraction of players selecting w is also higher than the number of pessimistic types but there is no impact in the payoffs resulting from each action.

Now, given that states MP and P result in bankruptcy, the assessed probability of bankruptcy by an optimistic player that did not receive a message is $\frac{1}{10} + \frac{1}{5} = 3/10$. Given the assumption on the parameter values, this implies that given this information set, r is an optimal action as long as $\frac{d}{z} \geq \frac{3}{7}$. In this way, for $\frac{d}{z} \geq \frac{3}{7}$, the candidate equilibrium strategy satisfies: $s^o = \{0, 1, w, r, a_{pr}^o, r, w, w\}$.

The expected utility for an optimistic player that does not acquire information is:

$$V(0; s^o, o) = \Pr(O \text{ or } MO | t_i = o)(1 + d)$$

$$V(0; s^o, o) = \frac{7}{10} + d\frac{7}{10}$$

Applying Bayes' rule, it can be verified that after an optimistic type receives message r, the resulting subjective probability of bankruptcy is equal to 1/6. Similarly, after an optimistic type receives message r, the resulting subjective probability of default equals 1/2. Using this information, some simple algebra shows that the postulated second stage actions for an optimistic agent that acquires information are optimal as long as $\frac{1}{5} \leq \frac{d}{z} \leq 1$. These two conditions are satisfied for the range of parameter values considered in this case. Additionally, as long as $\frac{d}{z} \leq 1$ a pessimistic type that receives message r would still find that w is the optimal second stage actions and, hence, the play is no contingent on the signal, that is, the signal has no value.

The expected utility for an optimistic player that acquires information is:

$$
\begin{aligned}
V(1; s^o, o) \quad &= \Pr(P|t_i = o)\Pr(m = w|P)z \\
&+ \Pr(MP|t_i = o)\Pr(m = w|MP)z \\
&+ \Pr(MO|t_i = o)\left[\Pr(m = w|MO)\right. \\
&+ \Pr(m = r|MO)(1 + d)\right] + \Pr(O|t_i = o)\left[\Pr(m = w|O)\right. \\
&+ \Pr(m = r|O)(1 + d)\right]
\end{aligned}
$$

$$
V(1; s^o, o) = \frac{1}{10}\frac{4}{5}z + \frac{1}{5}\frac{3}{5}z + \frac{3}{10}\left[\frac{2}{5} + \frac{3}{5}(1 + d)\right] + \frac{2}{5}\left[\frac{1}{5} + \frac{4}{5}(1 + d)\right]
$$

$$
V(1; s^o, o) = \frac{1}{5}z + \frac{7}{10} + \frac{1}{2}d
$$

The value of information is given by:

$$
V(1; s^o, o) - V(0; s^o, o) = \frac{1}{5}z - \frac{1}{5}d
$$

Hence, for $\frac{d}{z} \geq \frac{3}{7}$, s^o is an equilibrium as long as the value of information is higher than c.

For the case $\frac{d}{z} < \frac{3}{7}$, an optimistic that does not acquire information finds that the optimal action in the second stage is w. This alters the value of information since, for this case, the expected utility of an optimistic agent that does not acquire information equals:

$$
V(0; s^o, o) = \Pr(O \text{ or } MO|t_i = o) + (1 - \Pr(O \text{ or } MO|t_i = o))z
$$

$$
V(0; s^o, o) = \frac{7}{10} + z\frac{3}{10}
$$

Since the utility associated to collecting information is the same as in the previous case, we have that the value of information is given by:

$$
V(1; s^o, o) - V(0; s^o, o) = \frac{1}{2}d - \frac{1}{10}z
$$

Hence, for $\frac{d}{z} < \frac{3}{7}$, s^o is an equilibrium as long as the value of information expressed above is higher than c. The lemma results from combining the conditions of the two cases under consideration. □

Hence, for this type of equilibria to exist, the gains from placing a deposit in a good state need to be sufficiently high and the cost of information needs to be sufficiently low compared to the value of information. The condition for equilibrium reflects that the value of information takes two different forms depending on the preferred action by an optimistic player that does not collect information but knows that other optimistic players are collecting information.

Given the result of lemma 1, it is of interest to check the conditions such that there exist values of the cost parameter that allow for multiple equilibria. This is the case when the lower bound on c impose by lemma 1 is below the upper bound required by lemma 2. The following proposition shows that the value of information is these scenarios changes in a way such that multiple equilibria can re-emerge for an ample set of the original game payoff parameters.

Proposition 1:

For $\frac{1}{5}z \le d \le \frac{2}{3}z$, there exist a nonempty range of values for c such that the game has both an equilibrium with no acquisition of information and an equilibrium in which optimistic types acquire information.

Proof:

Three cases must be considered since different conditions operate in each case. First, for $\frac{1}{5}z \le d \le \frac{1}{4}z$, there is an equilibrium with no information acquisition as long as $c \ge c^l = \frac{2}{25}z - \frac{8}{25}d$ and there is an equilibrium in which optimistic players acquire information if $c \le c^h = \frac{1}{2}d - \frac{1}{10}z$. It is easy to verify that, for this case, $c^l < c^h$ then, there exist a nonempty set of values for c such that both equilibria exist. Next, for $\frac{1}{4}z \le d \le \frac{3}{7}z$, there is an equilibrium with no information acquisition as long as $c \ge c^l = \frac{8}{25}d - \frac{2}{25}z$ and there is an equilibrium in which optimistic players acquire information if

$c \leq c^h = \frac{1}{2}d - \frac{1}{10}z$. It can be shown that for this case, $c^l < c^h$, there exist a nonempty set of values for c such that both equilibria exist. Finally, for $\frac{3}{7}z \leq d \leq \frac{2}{3}z$, there is an equilibrium with no information acquisition as long as $c \geq c^l = \frac{8}{25}d - \frac{2}{25}z$ and there is an equilibrium in which optimistic players acquire information if $c \leq c^h = \frac{1}{5}z - \frac{1}{5}d$. Again, since $c^l < c^h$, there exist a nonempty set of values for c such that both equilibria exist. □

This result shows that allowing for acquisition of information regarding other beliefs, reintroduces equilibrium indeterminacy commonly observed in analysis of bank run scenarios. In addition, it is shown that information acquisition can increase the probability of a successful run. If optimistic players acquire information, bankruptcy will be observed not only in the "pessimistic" state but also in the "medium pessimistic" state. This implies that, for the scenario under analysis, information acquisition can have harmful effects and a raise in the cost associated to this activity can have beneficial effects.

We provide results on the existence of equilibria in which pessimistic types acquire information. First, it is shown that, in this set up, strategic complementarities are not sufficiently strong so that both types of players acquire information in equilibrium. Second, it is shown that for sufficiently strong fundamentals there is no equilibrium in which pessimistic types acquire information.

Proposition 2:

With costly information, there are no symmetric equlibria involving information acquisition by both types of players. Additionally, if $L > 16/25$, there are no symmetric equilibria in which only pessimistic types acquire information.

Proof:

There are no symmetric equilibria in which pessimistic and optimistic types do research. Note that if all players do research it needs to be the case that the information is valuable for all. That is, all players select the action indicated by the message. It is easy to show that the updated beliefs of a pessimistic type

that receives message r and an optimistic type that receives message w are the same. But in equilibrium their actions must be different. For that to be optimal, it must be the case that the expected payoffs associated to each action coincide. But then, information is not valuable and information acquisition by both types cannot be an equilibrium.

Finally, for $L > 16/25$, research by pessimistic type cannot be an equilibrium. Note that if pessimistic types do research then, it should be the case and research is valuable and, in the equilibrium path, second stage action selection coincides with the message perceived. Then, in state $P, 1/5$ of the pessimistic types receive message r and, as a result, choose r in the second stage. Hence, in this state, the fraction of agents that select w is $\frac{4}{5} - \frac{4}{5}\frac{1}{5} = \frac{16}{25}$ and no bankruptcy occurs in the equilibrium path. Then, there is no value of information. $\square$

An equilibrium in which pessimistic players acquire information does not exist for an ample subset of the values of the parameter L that controls the threshold of withdrawals above which the bankruptcy occurs. This is because the fraction of pessimistic players that would receive signal r and, as a consequence, would find it optimal to select "renew" is such that, for a relatively high value of L, bankruptcy would not occur in any state. Naturally, for sufficiently low levels of L an equilibrium in which pessimistic players acquire information is not ruled out.

The previous proposition indicates that strategic complementarities in information acquisition are not strong enough so that there exists an equilibrium in which all players acquire information. The next section considers a change in the technology of information acquisition such that this type of equilibrium can exist.

IV.3. Prominent player

So far we have assumed that, once we condition on the state, the information acquired is not correlated. But, given the initial information structure, the set of equilibria and the outcome of the game could be different

under different properties of acquired information. For example, the diversity in the information acquired, can be influenced by the shape of the network through which information is channeled. In this subsection we consider a scenario in which signals are highly correlated. More specifically, we assume that the signal observed by all players that acquire information, equals the tentative action of a "prominent" player that is selected with equal probability among all players participating in the game. This is an extreme situation in which the diversity in the content of signals is driven to a minimum. We use this case to illustrate the impact of changes of technology of information transmission on the incentives to acquire information and the associated equilibria of the game.

Proposition 3:

With perfectly correlated signals and $c \leq \frac{2}{5}d$, there exist an equilibrium in which all players acquire information. Additionally, if $\frac{16}{50}d \leq c \leq \frac{29}{50}d$ there exists an equilibrium in which optimistic players acquire information.

Proof:

Under perfect correlation of the signal, when all players acquire information, the assessed probability of bankruptcy is equal to the probability that the signal is w, which is equal to the proportion of pessimistic players. This proportion can be computed as the sum of the probability of each state times the proportion of pessimistic players in each state. Straightforward computations show that for a pessimistic player this number equals 3/5 and for an optimistic player this number equals 2/3. Then, as a consequence of the perfect coordination when all players acquire information, the expected payoff for an optimistic type is $\frac{2}{5}z + \frac{3}{5}(1 + d)$ and the expected payoff for a pessimistic type is $\frac{3}{5}z + \frac{2}{5}(1 + d)$. The value of information is given by the difference between these expected payoffs and the expected payoffs when no information is acquired. As long as $\frac{d}{z} < \frac{2}{3}$, a player that does not acquire information (optimistic or pessimistic) finds it optimal to choose "withdraw". Hence the expected payoffs are $\frac{3}{5} + \frac{2}{5}z$ for an optimistic type and $\frac{2}{5} + \frac{3}{5}z$ for a

pessimistic type and the value of information is $\frac{3}{5}d$ for an optimistic type and $\frac{2}{5}d$ for a pessimistic type. Note that, since the default action is "withdraw", the value of information is lower for the pessimistic type. Then, there exist an equilibrium in which all players acquire information as long as $c \leq \frac{2}{5}d$.

Under perfect correlation of the signal, when optimistic players acquire information, the assessed probability of bankruptcy is equal to the probability that the signal is w, which is equal to the proportion of pessimistic players. This proportion can be computed as the sum of the probability of each state times the proportion of pessimistic players in each state. Straightforward computations show that for an optimistic player this number equals $21/50$. Then, as a consequence of the perfect coordination when all players acquire information, the expected payoff for an optimistic type is $\frac{21}{50}z + \frac{29}{50}(1+d)$. The value of information is given by the difference between these expected payoffs and the expected payoffs when no information is acquired. As long as $\frac{d}{z} < \frac{2}{3}$, a player that does not acquire information (optimistic or pessimistic) finds it optimal to choose "withdraw". Hence the expected payoffs are $\frac{29}{50} + \frac{21}{50}z$ for an optimistic type. Then, an optimistic type finds it optimal to acquire information if $c \leq \frac{29}{50}d$. Similar calculations show that, in this case, a pessimistic type would find that not acquiring information is a best response as long as $c \leq \frac{16}{50}d$. $\square$

As in the case in which signals are not correlated, equilibria with information acquisition are associated to a probability of default that is higher than the probability of default in the case in which no information is acquired. Also we can verify, that under correlated signals, these type of equilibria exist under weaker assumptions.

Corollary 1:

Under perfectly correlated signals, the range of parameter such that there exists an equilibrium in which optimistic players acquire information is larger

than the corresponding range of parameters of the game with independent signals.

Proof:

In the proof of Proposition 3, it is shown that as long as $c \leq \frac{29}{50}d$ there exists and equilibrium in which optimistic players acquire information. This condition on the parameters can be shown to be weaker that the conditions required for this type of equilibrium in the case in which signals are independent. Note that for $d \leq \frac{3}{7}z$, lemma 2 requires $c \leq \frac{1}{2}d - \frac{1}{10}z$ which is smaller than the upper bound of proposition 3. In addition, for $d > \frac{3}{7}z$, the condition of lemma 2 is $c \leq \frac{1}{5}z - \frac{1}{5}d$. Replacing z by the smaller number $\frac{7}{3}d$, we get $c \leq \frac{1}{5}\left(\frac{7}{3}d\right) - \frac{1}{5}d = \frac{4}{15}d$ which results in another upper bound that is smaller than the upper bound from proposition 3. □

The scenario under analysis constitutes one instance in which easier coordination through correlated information implies that worse equilibrium outcomes are possible. This result has implications not only about the desirability of information transmission but also about the convenience of different technologies of information transmission.

The following proposition establishes the result on equilibrium indeterminacy for the case of correlated signals.

Proposition 4:

With perfectly correlated signals there always exists a nonempty range of values $[c^l, c^h]$ such that if c belongs to that range, there exist an equilibrium in which all players acquire information and an equilibrium in which no player acquires information.

Proof:

The conditions for an equilibrium in which no player acquires information are the same as in the original game. This is because if no player acquires

information, the correlation in the signals does not have an impact on the relative payoffs of an agent that is assessing the benefits of acquiring information. As shown in lemma 1, for $\frac{d}{z} \geq \frac{1}{4}$ the condition for this type of equilibrium is $c \geq \frac{8}{25}d - \frac{2}{25}z$ and for $\frac{d}{z} \leq \frac{1}{4}$ the condition for this type of equilibrium is $c \geq \frac{2}{25}z - \frac{8}{25}d$.

We need to consider conditions such that both equilibria exist. Remember that in proposition 3, we verified that there is an equilibrium in which all players acquire information as long as $c \leq \frac{2}{5}d$. For $\frac{d}{z} \leq \frac{1}{4}$, the conditions that must be satisfied for the existence of both equilibria are $c \geq c^l = \frac{2}{25}z - \frac{8}{25}d$ and $c \leq c^h = \frac{2}{5}d$. Since the initial parameter restrictions includes $\frac{d}{z} \leq \frac{2}{3}$, it can be shown that $c^l \leq c^h$. For $\frac{d}{z} \geq \frac{1}{4}$, the conditions that must be satisfied for the existence of both equilibria are $c \geq c^l = \frac{8}{25}d - \frac{2}{25}z$ and $c \leq c^h = \frac{2}{5}d$. Since the initial parameter restrictions includes $\frac{d}{z} \geq \frac{1}{9}$, it can be shown that $c^l \leq c^h$. Hence, in both cases there exist values for parameter c such that both types of equilibria exist. □

V. Concluding remarks

We have analyzed a simple game of bank runs. We have argued that, for first-play type of situations insights can be gained by considering a scenario in which players have a tentative action profile where actions are non-consistent and can be revised by performing activities that permit learning about others' tentative play. We identify conditions under which equilibrium indeterminacy re-emerges in the game with information acquisition. With respect to the second issue, for the configuration under analysis, the equilibrium with positive levels of information acquisition results in higher probability of bankruptcy. In addition, we show that correlated signals enlarge the set of parameter values such that there exist equilibria with positive levels of information acquisition and higher probability of bankruptcy. These results have implications for the convenience of different forms of information transmission in circumstances of strategic uncertainty. They suggest that

communication can be harmful and correlated messages can be worse than independent messages.

This analysis naturally demands empirical exercises to evaluate the fitness of different predictions. Experiments seem to be the most natural environment where coordination and research efforts by participants playing the game on a first occasion can be evaluated.

Another direction in which our analysis can be further developed has to do with exploring richer dynamics. Throughout this work we have emphasized that first-play type of situations is the environment that we have in mind. But the approach we have developed could be evaluated jointly with adaptive learning assumptions. Non-stationary conditions in particular contexts of repeated interaction where payoffs can change abruptly are definitely other settings where perspectives similar to the ones we developed in this work could be applied.

Finally, two other areas in which this type of analysis can be further developed are the consideration of public signals and diversity in the level of sophistication of players. Public signals can play an important role in facilitating coordination. We have partly addressed this issue by allowing correlation in the research signals. Regarding levels of sophistication, we have assumed that all players use a similar type of simple rule to select actions. More sophisticated rules would involve further inferences regarding the value of an action given the rules used by other players. We could consider a situation in which cognitively constrained agents have to choose between allocating resources to inner deliberations or to research about others' intention.

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