Power System Linear Modeling. Application for Small Signal Stability Analysis

Carlos E. Biteznik, Jorge L. Agüero y Mario C. Beroqui

Instituto de Investigaciones Tecnológicas para Redes y Equipos Eléctricos - Laboratorio de Alta Tensión.
Facultad de Ingeniería - Universidad Nacional de La Plata
Calle 48 y 116, B1900AMF, La Plata, Argentina
Telef:+54 221 4250804, fax:+54 221 4250804 int. 322, e-mail: cebiteznik@iitree-unlp.org.ar, jla@iitree-unlp.org.ar, mberoqui@iitree-unlp.org.ar

Abstract

This paper presents the calculation of the state matrix *A* of the power system through a program developed under MATLAB. This program is called SIAM (*Sistema Informático para Análisis Modal - Computational System for Modal Analysis*). The state matrix is obtained from the load flow solution data and from the dynamic model parameters employed. The detailed models of synchronous machines, automatic voltage regulators and power system stabilizers are linearized for the calculation. To evaluate the method proposed, there is a comparison of results between SIAM and a commercial program for a reference case published.

Keywords

Inter-area oscillation, Modal Analysis, Mode Oscillation, Power system stability, Small Signal Stability.

1. Introduction

The modal Analysis determines the eigenvalues or modes of oscillation of the system, expressed in terms of their frequency and damping. For modal analysis, the system state matrix *A* is required. This matrix is composed by the partial derivatives of the state variables referred to each other [1].

In studies carried out over the Argentinean interconnected power system (SADI-SIP), the state matrix is obtained from the same nonlinear models used for transient stability studies [2]-[3]. The dynamic model database is managed by the transmission system operator (CAMMESA) and is available only for the commercial program PSS/E (hereinafter referred as Commercial Program - CP). This implies some limitations to perform specific analysis on the power system.

For more flexibility, a program under MATLAB was developed to build and to process the state matrix, instead of the specific module on the CP package. This program was called SIAM.

SIAM was developed to obtain a more versatile tool for studies of small signal stability. This tool can be used not only to calculate the eigenvalues and eigenvectors, but also to obtain the participation factors, mode shape, controllability and observability indices such as the residues. These features allow the design of different control strategies to avoid instabilities.

This program has the additional advantage of being able to perform the analysis based on a load flow solution without the need to migrate all data to any new software. In this case it is used a specific CP, but it could be used with any other. Additionally to the load flow solution data, it only needs to know the dynamic model parameters employed. The SIAM contains more than 100 types of standard and user developed models to represent the synchronous machine (SM), Automatic Voltage Regulator (AVR) and Power System Stabilizer (PSS).

Another advantage of the SIAM is the analytical method of derivative calculation, which allows the use of highly nonlinear models with a bounded mistake. When the incremental method is used (step-type disturbance on the state variables for the derivative computation), there are greater errors which could spoil the analysis.

2. Differential-Algebraic Model

A. Description

The dynamic behavior of a power system can be described by a set of nonlinear differential equations called Differential-Algebraic Equations (DAE) shown in (1):

$$\begin{aligned} \dot{x}_d &= f\left(x_d, x_a, u\right) \\ 0 &= g\left(x_d, x_a, u\right) \\ y &= h\left(x_d, x_a, u\right) \end{aligned} \tag{1}$$

where *f* represents the dynamic characteristic of the system components, while *g* represents the nonlinear network equations. The state variables x_d belong to generator models and control elements in the system, such as AVR and PSS. For this analysis, the algebraic variables x_a are the injected currents ($\hat{I}_g$) by the generators and the voltages on each node ($\hat{U}=Ue^{j\theta}$).

The *h* function represents the output behavior and *u* are the independent inputs. Both, *h* and *u* are considered null for this first analysis. At the same time, *g* consists of two functions, one describing the link between the generator stator and the grid (g_1) and another corresponding to the relations between network nodes (g_2). These two functions are call Stator Equations and Network

Equations respectively [4]. With these assumptions, (1) can be described as (2):

$$\begin{aligned} \dot{x} &= f\left(x, \hat{I}_g, \hat{U}\right) \\ 0 &= g_1\left(x, \hat{I}_g, \hat{U}\right) \\ 0 &= g_2\left(x, \hat{I}_g, \hat{U}\right) \end{aligned} \qquad (2)$$

were g_1 and g_2 are given for the *i-th* bus by:

$$\begin{aligned} g_{1i} &= U_i e^{j\theta_i} - \left(-\psi''_{qi} + j\psi''_{di}\right)\omega_i e^{j\left(\delta_i - \pi/2\right)} + \\ &+ \left(R_i + jX''_i\right)\left(I_{di} + jI_{qi}\right) e^{j\left(\delta_i - \pi/2\right)} = 0 \end{aligned} \qquad (3)$$

$$\begin{aligned} g_{2i} &= U_i e^{j\theta_i}\left(I_{di} - jI_{qi}\right) e^{-j\left(\delta_i - \pi/2\right)} + P_{Li} + jQ_{Li} - \\ &- \sum_{k=1}^{n} U_i e^{j\theta_i} U_k e^{-j\theta_k} Y_{ik} e^{-j\alpha_{ik}} = 0 \end{aligned} \qquad (4)$$

where ψ''_d and ψ''_q are the *dq* components of sub-transient flux, ω is the rotor speed, R is the stator resistance, X'' is the sub-transient reactance, I_d and I_q are the *dq* components of the current, δ is the rotor angle, $P_{Gi}+jQ_{Gi} = U_i e^{j\theta i}(I_{di}-jI_{qi})e^{-j(\delta i-\pi/2)}$ is the power injected by the generator in the *i-th* bus, $Y_{ik}e^{j\alpha ik}$ is the *i-k* element from the admittance matrix of the system, and n is the number of buses. P_{Li} and Q_{Li} are the active and reactive load power demanded in the *i-th* bus, which could be nonlinear functions of the bus voltage.

B. System Linearization

If (2) is linearized for a given operating point, and written in matrix form, the DAE full matrix result:

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_1 & D_2 \\ C_2 & D_3 & D_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta\hat{I}_g \\ \Delta\hat{U} \end{bmatrix} \qquad (5)$$

Full matrix is formed by sub-matrices. The first row of (5) represents the linearized expression of the *f* funtion from (2). The sub-matrices A, B_1 and B_2 describe how the state variables and algebraic variables affect the derivatives for each state variable. Those are computed analytically based on each specific model. At the same time, each of these matrices is composed of *m* sub-matrices, independent from each other, which represent the linearized models used for the dynamic representation of each plant [4]-[6]. A and B_1 are diagonal matrices while B_2 is an sparse matrix. The sub-matrices shape of A, B_1 and B_2 are shown in (6).

$$A_i = \begin{bmatrix} A_{GENi} & A_{GENi-AVRi} & A_{GENi-PSSi} \\ A_{AVRi-GENi} & A_{AVRi} & A_{AVRi-PSSi} \\ A_{PSSi-GENi} & A_{PSSi-AVRi} & A_{PSSi} \end{bmatrix}$$

$$B_{1\,i} = \begin{bmatrix} B_{1_{GENi}} \\ B_{1_{AVRi}} \\ B_{1_{PSSi}} \end{bmatrix} \quad B_{2\,i} = \begin{bmatrix} B_{2_{GENi}} \\ B_{2_{AVRi}} \\ B_{2_{PSSi}} \end{bmatrix} \qquad (6)$$

The second and third row of (5) represent the linearized expressions of the functions *g1* and *g2* from (2), respectively. The matrices C_1 and D_1 are diagonal while C_2, D_3 and D_2 are sparse matrices. The matrix D_4 is known as the Jacobian of the network and its elements are the derivative expressions of the active and reactive power, respect to the module and voltage angle.

The system state matrix (A_{syst}) is obtained from (5) by eliminating the algebraic variables. The mathematic model is reduced to (7):

$$\Delta\dot{x} = A_{syst}\Delta x \qquad (7)$$

The modal analysis is performed based on the system state matrix (A_{syst}) from which the eigenvalues (λ) and eigenvectors (right eigenvector V and left eigenvector W) can be calculated.

3. Computational Implementation

In order to calculate the coefficients of the different sub-matrices, it is necessary to have information regarding the load flow solution of the case under analysis. These results are obtained in the CP environment. Prior to data collection, all generators must be converted to current sources with output impedance equal to the stator impedance. The loads must be converted, considering the voltage value of the node, to constant admittance, constant current or constant power loads in whichever proportion.

The general structure of the calculation method of the DAE full matrix is described in Figure 1.

Data from the CP is entered to SIAM. In the first step the program sorts the data from the load flow solution and incorporates the dynamic model parameters for each SM and the control elements, such as AVR and PSS. At this point, SIAM already has all data necessary for

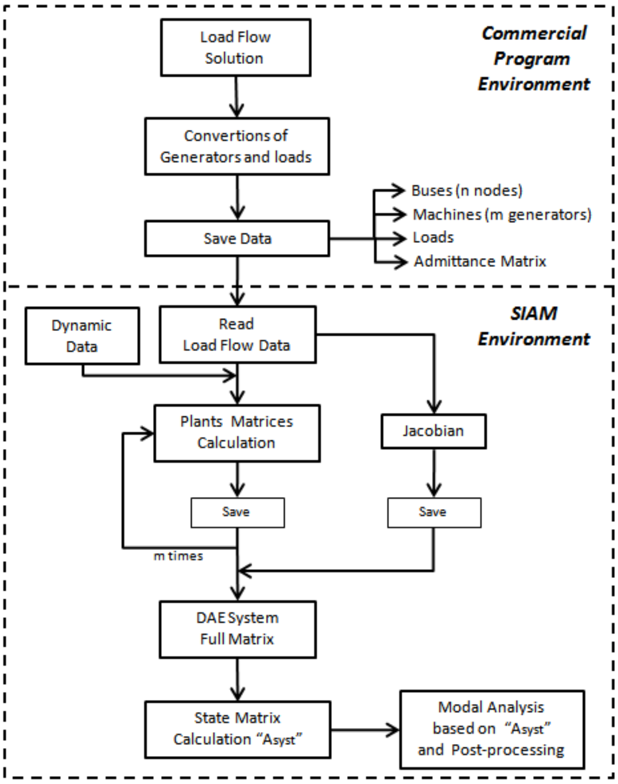

Figure 1: General structure of the calculation method.

constructing the DAE full matrix. The assembly of this is done by computing the sub-matrices: A, B_1, B_2, C_1, C_2, D_1, D_2, D_3 corresponding to the m generation buses. Also, the Jacobian (matrix D_4) of the network is calculated.

When the DAE full matrix is ready, the state matrix (A_{syst}) is obtained from this. The eigenvalues (λ) and eigenvectors are calculated, as well as the frequency and damping of the modes obtained.

4. Mode Shape, Participation Factors and Controllability Index

A. Mode shape

The right eigenvector v_i is known as mode shape corresponding to λ_i. For a given eigenvalue, the mode shape is very useful for identifying a group of coherent generators in a multi-machine system [1], [6].

B. Participation Factor

Any arbitrary element v_{ki} in V can be seen as contribution of the *i-th* mode in the *k-th* state variable, i.e. activity of the *i-th* mode in the *k-th* state variable. On the other hand w_{ik} corresponds to the weight of the contribution of the *k-th* sate variable to *i-th* mode. The product of v_{ik} and w_{ki} is, however, a dimensionless measure known as participation factor [1]. The most generic definition of participation factor is given as:

$$p_{ki} = |v_{ki}||w_{ik}| \tag{8}$$

C. Controllability index

When the system has independent input such as FACTS or other devices outputs [6]-[7], the reduced model (7) result:

$$\Delta\dot{x} = A_{syst}\Delta x + F\Delta u \tag{9}$$

Substituting $\Delta x=V\Delta z$ in (9), result:

$$\Delta\dot{z} = WA_{syst}V\Delta z + WF\Delta u \tag{10}$$

Expression (10) can be written for the *k-th* mode as:

$$\Delta\dot{z}_k = \lambda_k\Delta z_k + \sum_{i=1}^{l} w_k^T f_i \Delta u_i \tag{11}$$

where w_k is the left eigenvector corresponding to the *k-th* mode, f_i is the *i-th* column vector of matrix F and l is the total number of independent inputs. The controllability index (CI) of the *i-th* input to the *k-th* mode is defined as:

$$CI_i = w_k^T f_i \tag{12}$$

For the case of an SVC, with susceptance B_{svc}, which is placed at bus i, the reactive power injected into de bus (Q_{svc}), the most basic model is given by:

$$Q_{svci} = B_{svci}U_i^2 \tag{13}$$

Considering only variations of B_{svci} and linearizing (13), it is obtained:

$$\Delta Q_{svci} = U_i^2\Delta B_{svci} \tag{14}$$

If (5) is reduced by elimination of the algebraic variable $\hat{I}_g$, it can be rewritten as:

$$\begin{aligned} \Delta\dot{x} &= A\Delta x + B\Delta\hat{U} \\ 0 &= C\Delta x + D\Delta\hat{U} \end{aligned} \tag{15}$$

The second equation of (15) represents the power balances of the system, combining this with (14) provides:

$$0 = C\Delta x + D\Delta\hat{U} + H\Delta B_{svc} \tag{16}$$

where matrix H contains partial derivatives of power balances equation at all the buses with respect to the susceptance of the SVC. Substituting (16) into (15) and eliminating the algebraic variables $\Delta\hat{U}$, result:

$$\begin{aligned} \Delta\dot{x} &= \left(A - BD^{-1}C\right)\Delta x + \left(-BD^{-1}H\right)\Delta B_{svc} \\ \Delta\dot{x} &= A'\Delta x + H'\Delta B_{svc} \end{aligned} \tag{17}$$

The controllability index of SVC placed at the *i-th* bus to the *k-th* mode can be defined as given in (12).

$$CI_i = w_k^T h'_i \tag{18}$$

where w_k is the left eigenvector of matrix A' corresponding to the *k-th* mode.

5. Numerical example

The effectiveness of the proposed method was tested on 16-machines and 68-buses system [5]-[6]. This is a reduced order equivalent of New England Transmission System (NETS) and the New York Power System (NYPS). There are nine generators in NETS area and three in NYPS area. The three neighboring utilities are represented as three equivalent large generators #14, #15 and #16. Single line diagram of this system is shown in Figure 2.

This system was analyzed using the proposed method and its results were compared with those obtained from the commercial program, which is widely recognized worldwide.

Each SM was modeled to have three damper windings and one field winding. The first eight generators have DC excitation, while machine #9 was equipped with fast excitation. Machine #9 has also a speed input PSS.

These model characteristics were implemented using the generator model GENROU (IEEE Generator Model 2.2) [8], DC exciter model IEEET1 (IEEE Type DC1A), static exciter model ESST1A (IEEE Type ST1A) and PSS model IEEEST (IEEE Type PSS1A), all from CP IEEE standard library [9]-[10].All loads are considered of constant power.

A. Oscillation modes

A 700 MW flow between NETS and NYPS was assumed. This was performed by adjusting the load and generation in both areas.

As it was explained before, the resolution of the load flow is performed in the CP environment.

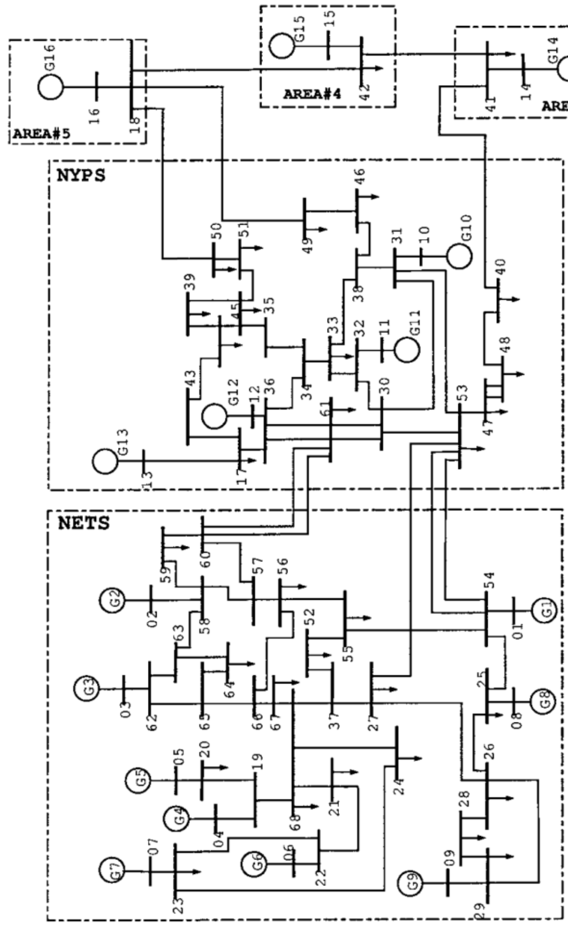

Figure 2. 16-machines and 68-buses system.

Subsequently, the state matrix was calculated by two methods, using SIAM and the activity ASTR through module program for dynamic studies from CP.

SIAM provides directly the eigenvalues of the system, while the matrix obtained by the CP must be post-processed using another module, which is included in the CP package.

The activity ASTR calculates the state matrix by applying a percentage perturbation to each state variable to obtain its derivative. When the models are nonlinear, this method is very sensitive to the amplitude chosen for the perturbation.

According to the experience obtained in different studies carried out over the Argentinean power system, a value of 1% results adequate. This value prevents large excursions of the state variables, or very small excursions that are lost in the numerical noise.

Figure 3 shows the eigenvalues obtained by the analytical method in the SIAM and by the incremental method in the CP.

The oscillation frequency and damping of the most important modes (damping < 10%) obtained by both programs are shown in Table I.

The greatest percentage error in frequency appear for mode 1 (1.49 %) and for damping in mode 2 (9.8 %). Despite the percentage damping error is high, the absolute error is just about 0.0047.

As it can be seen from the numerical example, when it is used the state matrix obtained from SIAM, the results obtained are similar to those achieved with the commercial program CP. The small differences in the results of both programs are due to the method used to obtain the state matrix, analytical versus incremental.

B. Participation Factor

The participation factors for all modes listed in Table I were computed using the expression (8). Also, the participation factors for the same modes were obtained from the CP. The results are shown in Table II. The values were normalized to the major.

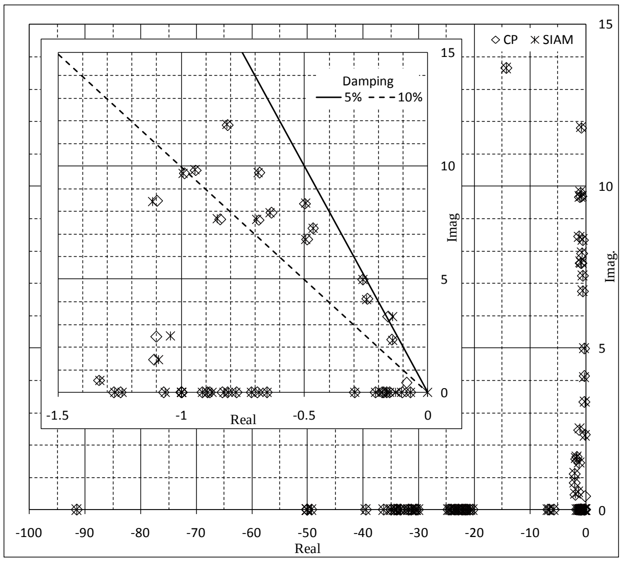

Figure 3. Eigenvalues obtained by both method.

Table I. Modes with damping ≤ 10%.

Mode	SIAM				CP			
	Real	Imag	Damping	Freq. (Hz)	Real	Imag	Damping	Freq. (Hz)
1	-0.1422	±2.2934	0.0619	0.3650	-0.1465	±2.3281	0.0628	0.3705
2	-0.1439	±3.3439	0.0430	0.5322	-0.1596	±3.3394	0.0477	0.5315
3	-0.2494	±4.0917	0.0608	0.6512	-0.2454	±4.1312	0.0593	0.6575
4	-0.2627	±4.9838	0.0526	0.7932	-0.2645	±4.9827	0.0530	0.7930
5	-0.4957	±6.7708	0.0730	1.0776	-0.4892	±6.7494	0.0723	1.0742
6	-0.4650	±7.2444	0.0641	1.1530	-0.4661	±7.2331	0.0643	1.1512
7	-0.6940	±7.6313	0.0906	1.2146	-0.6868	±7.6159	0.0898	1.2121
8	-0.6400	±7.9420	0.0803	1.2640	-0.6346	±7.9228	0.0798	1.2610
9	-0.4972	±8.3489	0.0594	1.3288	-0.4991	±8.3337	0.0598	1.3264
10	-0.9935	±9.6801	0.1021	1.5406	-0.9882	±9.6646	0.1017	1.5382
11	-0.6877	±9.6951	0.0708	1.5430	-0.6837	±9.6876	0.0704	1.5418
12	-0.9437	±9.8245	0.0956	1.5636	-0.9408	±9.8098	0.0955	1.5613
13	-0.8142	±11.8188	0.0687	1.8810	-0.8119	±11.802	0.0686	1.8784

Table II: Participation Factors.

Mode	SIAM		CP	
	Participation Factor	Machine	Participation Factor	Machine
1	1.00, 0.69, 0.61, 0.35, 0.14	13, 15, 14, 16, 12	1.00, 0.58, 0.53, 0.30, 0.14	13, 15, 14, 16, 12
2	1.00, 0.69	16, 14	1.00, 0.69	16, 14
3	1.00, 0.49, 0.38, 0.36, 0.35, 0.35, 0.25, 0.15, 0.15	13, 6, 5, 7, 4, 3, 2, 9, 1	1.00, 0.53, 0.40, 0.39, 0.38, 0.38, 0.28, 0.17, 0.17	13, 6, 5, 7, 3, 4, 2, 9, 1
4	1.00, 0.43, 0.15	15, 14, 16	1.00, 0.43, 0.15	15, 14, 16
5	1.00, 0.97, 0.41, 0.28, 0.17, 0.15	2, 3, 5, 6, 4, 7	1.00, 0.97, 0.40, 0.29, 0.17, 0.16	2, 3, 5, 6, 4, 7
6	1.00, 0.15	12, 13	1.00, 0.15	12, 13
7	1.00, 0.78, 0.33, 0.18	5, 6, 7, 4	1.00, 0.78, 0.34, 0.19	5, 6, 7, 4
8	1.00, 0.98	2, 3	1.00, 0.98	3, 2
9	1.00, 0.17, 0.14, 0.10	10, 8, 1, 9	1.00, 0.17, 0.14, 0.10	10, 8, 1, 9
10	1.00, 0.63, 0.23, 0.14	7, 6, 4, 5	1.00, 0.63, 0.22, 0.14	7, 6, 4, 5
11	1.00, 0.93	1, 8	1.00, 0.92	1, 8
12	1.00, 0.38, 0.28	4, 5, 7	1.00, 0.38, 0.27	4, 5, 7
13	1.00	11	1.00	11

C. Mode shape

The mode shape of the mode 1 (0.365 Hz) was obtained from its right eigenvector. The eigenvector elements corresponding to machine angle are shown in Figure 4-a. This shows two clusters of generators oscillating against each other. The first group corresponds to all generators from NEST and NYPS areas and the second group corresponds to the equivalent generators #14, #15 and #16. This is an inter-area oscillation and the major participants are the machines #13, #15, #14 and #16 as is shown in Table II.

Figure 4-b shows the mode shape of the mode 5 (1.0776 Hz). The eigenvector elements corresponding to machine angle. There are two clusters of generators oscillating against each other. The first group corresponds to the generators #2 and #3 from NEST and the second group corresponds to the generators #4, #5, #6 and #7 from the same area. This is an inter-unit oscillation.

The mode shape of the mode 13 (1.881 Hz) was obtained from its right eigenvector. The eigenvector elements corresponding to machine angle are shown in Figure 4-c. This shows the #11 generator oscillating against the rest of the system. This is a local oscillation.

Observing the Table II and the mode shape of each mode it is possible to determine that: the modes 1, 2, 3,4 and 9 are inter-area oscillations, the modes 5, 6, 7, 8, 10, 11 and 12 are inter-unit oscillations and the mode 13 are local oscillations [11].

D. Controllability Index

The controllability indices are performed for all modes listed in the Table II. Those are computed for an SVC in all bus locations of the study system, which is normalized with respect to the highest index.

The highest indices for each mode are shown in the Table III. For mode 1, it can be seen that the highest index corresponds to the bus 40. This bus is the arrival of the tie line that links NYPS with Area 3. If the indices are analyzed, the bus 40 has higher indices too for mode 2 (2nd) and mode 4 (1st). This means that an SVC located in this bus could help to control these three modes.

Table III: Controllability Index for an SVC.

Mode	SIAM	
	Controllability Index	Bus
1	1.000, 0.837, 0.816, 0.785	40, 48, 50, 51
2	1.000, 0.842, 0.881	50, 40, 51
3	1.000, 0.920, 0.910, 0.903	64, 22, 23, 21
4	1.000, 0.784	40, 48
5	1.000, 0.928, 0.895, 0.841	2, 3, 64, 62
6	1.000	12
7	1.000, 0.979, 0.958, 0.944	6, 22, 23, 7
8	1.000, 0.973	2, 3
9	1.000, 0.834, 0.743	9, 29, 28
10	1.000	7
11	1.000, 0.734	8, 25
12	1.000	4
13	1.000, 0.870, 0.828	32, 11, 33

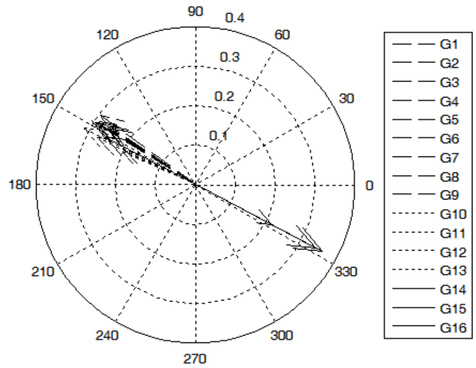

Figure 4-a. Mode shape of mode 1. Inter-area oscillations.

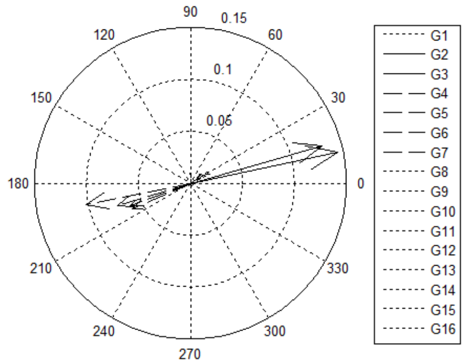

Figure 4-b. Mode shape of mode 5. Inter-unit oscillations

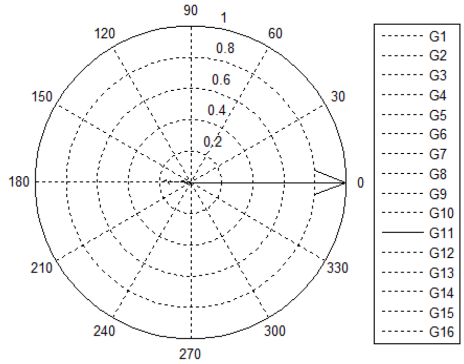

Figure 4-c. Mode shape of mode 13. Local oscillations

6. Conclusions

SIAM was developed to compute the DAE system full matrix of a power system and to obtain its eigenvalues and eigenvectors with this matrix.

SIAM only need as input the data from load flow solution and the dynamic model parameters. These inputs can be obtained from any commercial program used for stability studies.

SIAM was tested with a well known benchmark (16-machines and 68-buses system).

The differences between result obtained from a commercial program and SIAM are negligible, not only in the mode frequency but also in its damping.

Additionally, the availability of the DAE system full matrix in SIAM environment allows additional calculations such as controllability and observability indices. This availability is very important to formulate control strategies and to find optimal location of devices such as FACTS.

References

[1] P. Kundur, Power System Stability and Control. New York: McGraw- Hill, Inc. 1994. Chapter 12.

[2] J. Agüero, C. Biteznik et al, "Análisis Modal: Expansión del Sistema de Transmisión en 500 kV del SADI-SIP". XIV Encuentro Regional Ibero-Americano de CIGRE (ERIAC), Paraguay, May 2011.

[3] J. Agüero, C. Biteznik et al, "Linealización de Modelos del SADI-SIP para Análisis Modal". XIV Encuentro Regional Ibero-Americano de CIGRE (ERIAC), Paraguay, May 2011.

[4] P. Sauer, M. Pai., Power System Dynamics and Stability. New Jersey: Printice Hall. 1998. Chapter 6-8.

[5] B. Pal, B. Chaudhuri, Robust Control in Power Systems. USA: Springer 2005. Chapter 4.

[6] P. Zhang, C. Rehtanz, B. Pal, Flexible AC Transmission Systems: Modelling and Control. Berlin: Springer 2006. Chapter 12.

[7] B. Kalyan Kumar, S. N. Singh and S. C. Srivastava, "Placement of FACTS controllers using modal controllability indices to damp out power system oscillations", IET Gener. Transm. Distrib., Vol. 1, No 2, pp 209-217. March 2007.

[8] IEEE Guide for Synchoronous Generator Modeling Practices and Applications in Power System Stability Analyses. IEEE Std 1110-2002, Nov. 2003.

[9] PSS/E 30.2. Program Operation Manual. Volumen II. SIEMENS-PTI. 2005.

[10] IEEE Recommended Practice for Excitation System Models for Power System Stability Studies, IEEE Std. 421.5-2005, April. 2006.

[11] IEEE Guide for Identification, Testing, and Evaluation of the Dynamic Performance of Excitation Control Systems, IEEE Std. 421.2-1990, May. 1990.