A Mathematical Model of Ferromagnetic Behaviour Suitable for Simulation of Transient Phenomena in Electrical Power Systems

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ABSTRACT

The paper describes a conceptual model of ferromagnetic behaviour which, altough not based on physical grounds, becomes a useful "thinking device" for developing a computation algorithm suitable for being coupled to a general electromagnetic transients simulation programme such as EMTP.

A simple example is presented to show how this algorithm works in practice.

1.- Introduction

It is sometimes very important, for an accurate simulation of complex transient processes which arise in electrical power systems, to model in a faithful way the ferromagnetic behaviour of equipment such as transformers or saturable compensating reactors.

Although computer programs for digital simulation of transients, such as EMTP, often include options for good modelling of saturation (the classic Type 98 and Type 93 of EMTP) the hysteresis effects have a rather "rough" representation based on the limit hysteresis loop (Type 96 of EMTP). Such way of modelling hysteresis is in fact only accurate for calculation of transients which drive the saturable inductance alternating beyond the extreme saturation points, with rather ample oscillations. For transients which involve small oscillations of current (or flux) superimposed on a "pattern" of lower frequency and greater amplitude, this model will not give a good presentation of transient ferromagnetic hysteresis [1].

It may be that this is due, partly, because of lack of an understandable and conceptually correct theoretical model which can be developed into a simple algorithm with good performance in terms of accuracy, speed and memory requirements.

A theorical model which is rich in insight about the transient behaviour of ferromagnetism is the one described in [2].

But an algorithm based on this model appears to be cumbersome to couple to a general transients programme such as EMTP.

In this paper we develop another aproach which, although along a thinking line similar to that of [2], gives a mathematical model simpler to translate into a useful algorithm and yet shows an overall behaviour which represents very well that of ferromagnetism during complex transients.

Thereby, it is possible to develop a well structured and simple algorithm which follows, step by step, the state of the material and hence its macroscopic response, whatever the transient process in which it is involved.

The paper describes the proposed conceptual model, explains the theoretical background, develops the analytical justification of the algorithm and presents a simple example to show how it works in practice.

2.- The conceptual model

It is not intended that the model to be described is thought of as being related to any physical explanation; it is rather a "thinking device" used for developing a useful calculation procedure.

i) The ferromagnetic material is conceived as an aggregate of "bi-stable elements", with an idealized hysteresis loop such as is shown in Fig. 1.

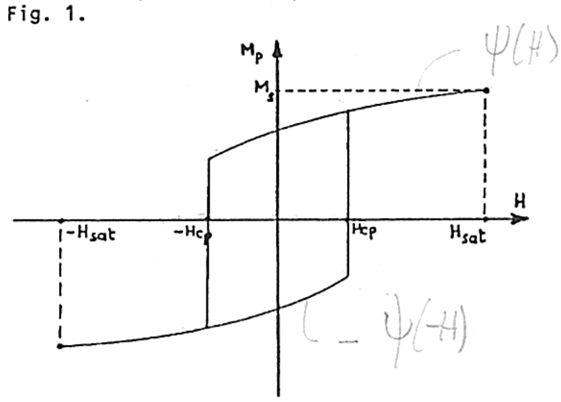

Fig. 1 - Elementary hysteresis cycle.

ii) The upper and lower branches of the hysteresis loop Mp vs. H of a given element p are described by the functional relationship.

$$Mp\ (H) = \begin{cases} \Psi(H) & \text{for } -Hcp < H \text{ (upper branch)} \\ -\Psi(-H) & \text{for } H < Hcp \text{ (lower branch)} \end{cases} \quad (1)$$

iii) The function $\Psi(h)$ is the same for all elements, being a characteristic of the material, positive and monotonically rising in the range $-H_{sat} < h < +H_{sat}$, where H_{sat} is the value of field intensity at which saturation of the material is assumed.

iv) The transition "jumps" between the upper and lower branches occur symmetrically at values of H = ±Hcp, the caracteristic "coercitivity" of element p. It is precisely the parameter Hcp which distinguishes the particular behaviour of element p from that of the others.

v) The infinitesimal elements have a uniform, homogeneous distribution in space, such that the elements contained in any arbitrarily small volume of material will always have the same distribution function of the characteristic parameter Hcp.

vi) Since the relationship between M_p and H is in part double valued, at any given external exciting field H the <u>state</u> of a given element p could be such that the corresponding value of M_p (H) may fall onto either of the branches of the hysteresis loop if $-Hcp < H < Hcp$, depending on the past evolution of H (history).

vii) The macroscopic magnetization M(H) of the bulk of ferromagnetic material will thus be the net value of the added contribution of the elements which, for the given H, are in the "up state" ($M_p = \Psi(H)$, with the same polarity than H), substracting the contribution of those which are in the "down state" ($M_p = -\Psi(-H)$, opposing the polarity of H).

3.- The mathematical model

The postulated conceptual model may be described mathematically as follows.

Let us considere, at time t, the relative quantity of elements contained in a unit volume for which the "next jump" in the hysteresis loop (trasition from one of the branches to the other) would occur at a value of H greater than a given h. The "actual state" of the material at time t will be described by the distribution function $v_t(h)$ of the fraction of unit volume having elements for which the "next jump" is greater or equal than h.

Since, because of changes of H, some elements will eventually switch " state" making their "next jump" to change sign, the distribution function $v_t(h)$ will be changing to follow the evolution of H(t).

Of course, $v_t(h)$ will always be a monotonically increasing function, from $v_t(h) = 0$, for $h \leq -H_{sat}$, up to $v_t(h) = 1$, for $h \geq H_{sat}$.

Known the above defined $v_t(h)$ as well as $\Psi(h)$, the macroscopic net value of magnetization M (H;t) will be given by the general formula:

$$M(H;t) = v_t(H).\Psi(H) - [1-v_t(H)].\Psi(-H) \qquad (2)$$

where H is the applied external field, assumed to be positive.

Starting at a reference time t_0, for which are known $H(t_0) = H_0$ and $M(H_0;t_0) = M_0$, the initial condition

$$M_0 = v_{t0}(H_0).\Psi(H_0) - [1-v_{t0}(H_0)].\Psi(-H_0) \qquad (3)$$

must be fullfilled.

Moreover, the effect of a change ΔH of H(t), from t to $t + \Delta t$, will be expresed by the incremental formula:

$$M(H+\Delta H;t+\Delta t) = M(H;t) + \Delta v_t(H).[\Psi(H)+\Psi(-H) +$$

$$+ v_t(H).\Delta\Psi(H) - [1 - v_t(H)]\Delta\Psi(-H) \qquad (4)$$

where

$$v_t(H) = v_t(H+\Delta H) - v_t(H)$$
$$(H) = \Psi(H+\Delta H) - \Psi(H)$$
$$(-H) = \Psi(-H-\Delta H) - \Psi(-H)$$

Accordingly, the distribution function $v_t(h)$ will change to $v_{t+\Delta t}(h)$ as follows:

$$v_{t+\Delta t}(h) = \begin{cases} v_t(h) & \text{for } h \leq -H-\Delta H \\ v_t(h) + \Delta v_t(H) & \text{for } -H-\Delta H < h < H+\Delta H \\ v_t(h) & \text{for } h \geq H+\Delta H \end{cases} \qquad (5)$$

Thus, the macroscopic behaviour of the ferromagnetic material can be calculated for any transient process using (3), (4) and (5), with a numerical step by step procedure.

4.- The characteristic functions of the model

To apply the model developed in the preceeding section, two characteristic functions of the material are needed: $\Psi(h)$ and $v_{t0}(h)$. Hence, a procedure should be found to calculate these functions from some practical data available for the particular ferromagnetic material considered. It will be shown that the suitable information for this is contained in the conventional <u>limit hysteresis cycle</u> of the material and its so-called <u>reversible permeability</u>.

Let us suppose that the material has been driven to saturation up to $H = -H_{sat}$. Thus, all elements will be in such a state (starting on the lower branch of the hysteresis loop) that then "next jump" will be at a positive Hcp. Let us call v2(h) the corresponding distribution fuction, which will have $v_2(h) = 0$ for $h \leq 0$, increasing up to $v_2(h) = 1$ for $h \geq +H_{sat}$.

If we <u>steadily increase</u> the field H, starting from $-H_{sat}$ and up to $+H_{sat}$, the "working point" will be sliding along the lower curve of the limit hysteresis cycle, which we named $M_2(H)$. Acording to our model, the magnetization observed macroscopically will be

$$M_2(H) = v_2(H).\Psi(H) - [1-v_2(H)].\Psi(-H) \qquad (6)$$

Recalling that we should have $v_2(H) = 0$ for $-H_{sat} \leq H \leq 0$, we find

$$M_2(H) = -\Psi(-H) \text{ for } -H_{sat} \leq H \leq 0 \qquad (7)$$

On the other hand, because of the symmetry of the limit hysteresis cycle, its upper curve es $M_1(H) = -M_2(-H)$. Therefore (7) may also be written

$$\Psi(H) = M_1(H), \text{ for } 0 \leq H \leq H_{sat} \qquad (8)$$

This defines part of the unknown function $\Psi(h)$.

Let us now suppose that at some value of $H > 0$ we <u>reduce H</u> by a very small ΔH, and that the observed reduction in magnetization is ΔM. Thus, by the definition of <u>the reversible permeability</u> $\mu_r(H)$, we will have

$$\mu_r(H).\Delta H \approx \Delta M = v_2(H).\underline{\Delta}\Psi(H) - [1-v_2(H)].\underline{\Delta}\Psi(-H) \qquad (9)$$

where
$$\underline{\Delta}\Psi(H) = \Psi(H) - \Psi(H-\Delta H)$$
$$\underline{\Delta}\Psi(-H) = \Psi(-H) - \Psi(-H+\Delta H)$$

We discretisize the independent variable H in a set of n suitably spaced points, thus: $H_0 = 0$, $H_1, \ldots, H_k \ldots, H_n$, $H_{n+1} = H_{sat}$.

Hence, combining (6), (8) and (9), we get the set of difference equations,

$$\left| \begin{array}{l} \underline{\Delta}\Psi(-H_k) = \dfrac{v_2(H_k).\underline{\Delta}M_1(H_k) - \mu_r(H_k).\Delta H}{1 - v_2(H_k)}, \\ \\ \Psi(-H_k) = \Psi(0) - \sum\limits_{i=1}^{k} \underline{\Delta}\Psi(-H_k), \qquad (10) \\ \\ v_2(H_k) = \dfrac{M_2(H_k) + \Psi(-H_k)}{M_1(H_k) + \Psi(-H_k)} ; \text{ for } k = 1,2,..,n \end{array} \right.$$

where

$$\underline{\Delta} M_1(H_k) = M_1(H_k) - M_1(H_k-\Delta H)$$

$$\underline{\Delta} \Psi(-H_k) = \Psi(-H_k) - \Psi(-H_k+\Delta H)$$

Since M_1, M_2 and μr are data, with (10), using a recursive procedure and starting with any reasonable gess for $v_2(h)$, e.g. a linear function, we can compute succcesive approximations to the required $v_2(h)$ and $\Psi(-h)$ at $h = H_k$ $(k = 1,\dots,n)$. This method is shown to converge very soon to the required accuracy (two or three terations).

If we had started with the material saturated at the positive Hsat, the characteristic volume distribution would be $v_1(h) = 1 - v_2(-h)$.

Furthermore, if the material is unmagnetized ("virgin"), it is reasonable to assume that the initial volume distribution is

$$v_0(h) = [v_1(h)+v_2(h)]/2$$

5.- Demostrating example

To prove the ability of the proposed model in reproducing the main features of ferromagnetic behaviour, a demostrating example has been taken from [3]. Although it is expressed in terms of induction B instead of magnetization M, the calculation model remains essentially the same.

Fig. 2 shows the limit hysteresis cycle with some "nearly reversible" minor loops for a sample of Permalloy 4-79.

From this, the approximate reversible permeability curve was drawn in Fig. 3.

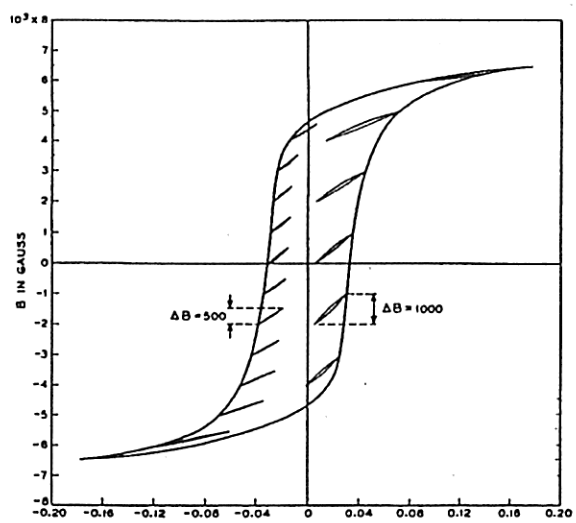

Fig. 2.-Minor and major hysteresis loops of 4-79 Permalloy. For the minor loops ΔB is constant at 500 (left side) or 1000 (right side) gauss. [3]

The method outlined in section 4. was applied using the numerical data taken from Fig. 2. Thus Fig. 4 shows the calculated $\Psi(h)$ along with $M_1(H)$ and $M_2(H)$; and Fig. 5 shows $v_2(h)$ and $v_0(h)$.

Using an algorithm based on the procedure presented in section 3., a transient was simulated consisting of oscillations of the exciting field H, with increasing amplitude, until full saturation is reached.

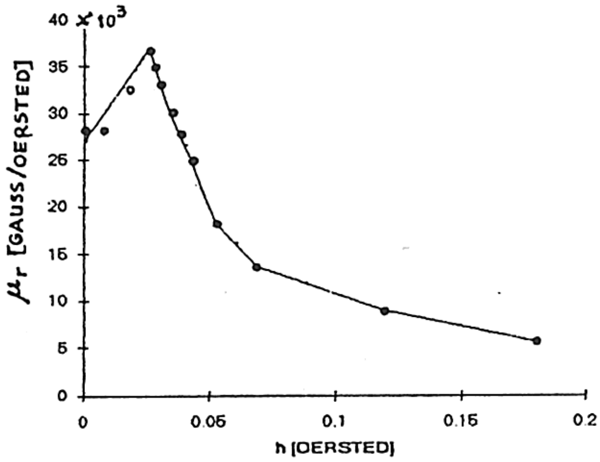

Fig.3 - Reversible permeability μr estimated for the sample of Permalloy 4-79 of Fig. 2 (• sample measured points).

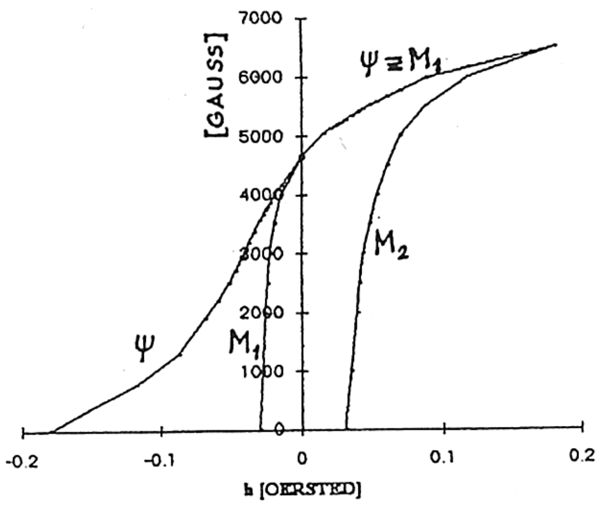

Fig.4 - Upper half of hysteresis cycle M_1, M_2 and characteristic function Ψ for the sample of Permalloy 4-79 of Fig. 2.

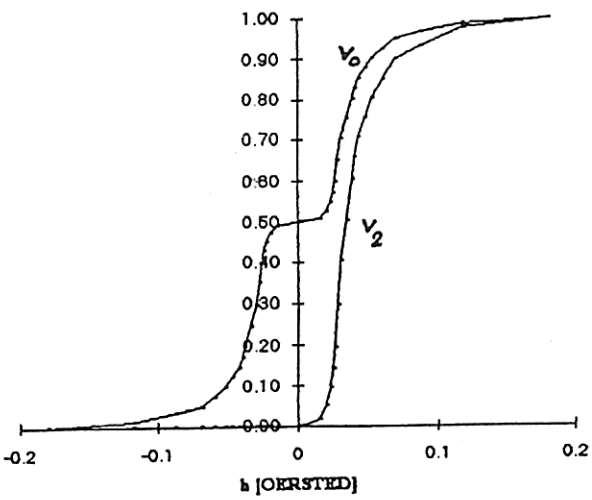

Fig.5 - Characteristic distribution v_0 for "virgin" and v_2 for saturated sample of Permalloy 4-79 of Fig. 2.

Fig. 6 shows the trajectory of the "macroscopic" magnetic state as it was calculated in the simulation.

Fig. 7 illustrates the evolution of the characteristic distribution v_t (h) for some relevant points of the macroscopic trajectory.

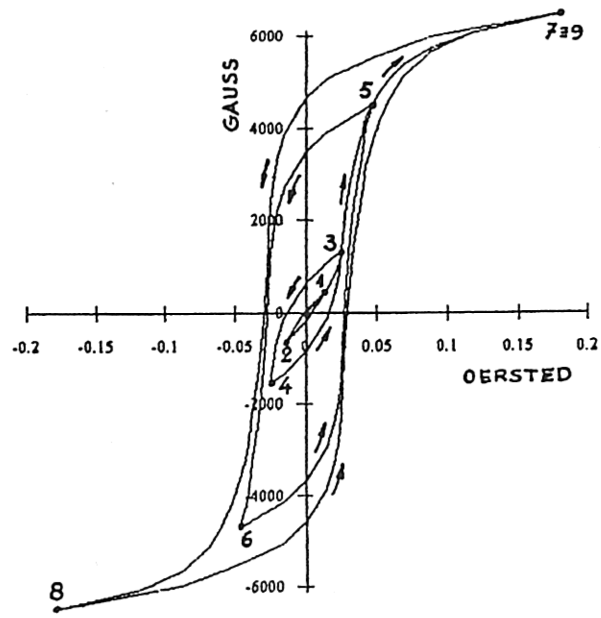

Fig.6 - Simulation of hysteresis loops of increasing amplitude for the sample of Permalloy 4-79 of Fig. 2. (1,2,...,9 points of field inversion).

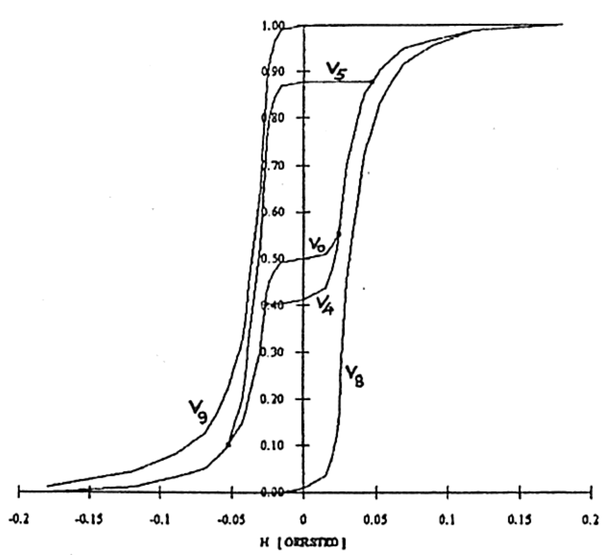

Fig.7 - Distributions which characterize the state of the material for the simulation of Fig. 6, at the initial (virgin) condition 0 and at the field inversion points 4,5,8 (negative saturation) and 9 (positive saturation).

6.- Conclusions

6.1.- A conceptual model has been presented which allows to build up a practical algorithm for the computation of ferromagnetic behaviour when simulating transient phenomena in electrical power systems.

6.2.- The ability of the algorithm has been demonstrated with a simple example.

6.3.- Further work is needed to couple this algorithm to a general computer program for the calculation of power system transients such as EMTP.

7.- References

[1] - Leuven EMTP Center: "Alternative Transients Program Rule Book". Leuven, 1987.

[2] - G. Biorci and D. Pescetti: "Analitycal theory of the behaviour of ferromagnetic materials". Il Nuovo Cimento, vol. VII, Nº 6, 1968, pp. 829-842.

[3] - R.M. Bozorth: "Ferromagnetism" (7th. printing). D. Van Nostrand Co., Inc., Princeton, New Jersey.